SRSTI 27.19.15

LINEAR MUTUALLY REDUCIBILITY DIFFERENTIAL EQUATIONS

ZH.A. SARTABANOV[1,*][0000-0003-2601-2678], **G.M. AITENOVA**[2][0000-0002-4572-8252], **G.S. TOREMURATOVA**[1][0000-0002-5601-0916]

[1]K. Zhubanov Aktobe Regional University, Aktobe, Kazakhstan

[2]West Kazakhstan State University. M. Utemisov, Uralsk, Kazakhstan

[*]e-mail: sartabanov42@mail.ru

Abstract. The article is devoted to the question of reducibility systems of ordinary differential equations, which originates from Lyapunov's classical work [1]. The idea of this research was developed by N.P. Erugin in [2]. An attractive direction in the theory of reducibility is the problem of reducibility linear systems with oscillatory coefficients, in particular, in the periodic case, we have the well-known Floquet theory [3].

It should be noted that the question of reducibility linear systems with variable coefficients to systems with constant coefficients is of particular interest.

In this research, this question is posed from a general point of view. If two linear systems with variable coefficients based on a non-singular linear replacement are reducible to each other, that is, mutually reducible, then the problem arises of clarifying the relationship that exists between these systems and their transformations.

The purpose of this article is to study this problem in some special cases and to establish a connection with known results by reducibility of systems of ordinary differential equations.

In connection with the solution of this problem, the statement of the question is given in the note. The question of the solvability of this problem is considered. The mutual reducibility of linear systems with periodic coefficients is investigated. A connection is established between the question of mutual reducibility and the Floquet theory, and the question of mutual reducibility of systems is investigated using the Lyapunov matrix, known from the theory of stability of motions.

In conclusion, it is noted that the results obtained in this work can be generalized in the case of linear systems of partial differential equations considered in [4-10].

Key words: linear system; mutual reducibility; matricant; transformation matrix; periodic system; Floquet's theorem; Lyapunov matrix; sustainability.

1. Mutual reducibility of linear equations in the class of continuous functions.

Consider the linear equations

$$\frac{dx}{d\tau} = A(\tau)x, \tag{1.1}$$

$$\frac{dy}{d\tau} = B(\tau)y \tag{1.2}$$

relatively n-vector functions x and y with continuous in $R = (-\infty, +\infty)$ with $n \times n$ matrices

$$A(\tau) \in C(R), \tag{1.3}$$

$$B(\tau) \in C(R) \tag{1.4}$$

where $C(R)$ is the class of continuous functions in R.

Suppose $T(\tau)$ is a nonsingular continuously differentiable $n \times n$ matrix:

$$\det T(\tau) \neq 0, \quad \tau \in R: \ T(\tau) \in C^{(1)}(R) \tag{1.5}$$

where $C^{(1)}(R)$ is the class of continuously differentiable functions, and the linear change

$$x = T(\tau) y \tag{1.6}$$

reduces equation (1.1) to equation (1.2).

To introduce the substitution (1.6) into equation (1.1), we define

$$\frac{dx}{d\tau} = \dot{T}(\tau) y + T(\tau) \frac{dy}{d\tau} \tag{1.6'}$$

where $\dot{T}(\tau) = \frac{dT(\tau)}{d\tau}$.

Substituting (1.6) and (1.6 ') into equation (1.1), we obtain the equation

$$\frac{dy}{d\tau} = T^{-1}(\tau)\left[A(\tau)T(\tau) - \dot{T}(\tau)\right]y \tag{1.7}$$

equivalent to equation (1.2).

Then, equating the coefficient of matrices of equations (1.2) and (1.7), we obtain the equation

$$\dot{T}(\tau) = A(\tau)T(\tau) - T(\tau)B(\tau) \tag{1.8}$$

for determine the transformation matrix $T(\tau)$ (1.6) satisfying conditions (1.5).

Obviously, if equations (1.1) and (1.2) have solutions $x(\tau)$ and $y(\tau)$ at $\tau \in R^{\circ} \subset R$, and hence $X(\tau)$ and $Y(\tau)$ are their nonsingular matrix solutions:

$$\dot{X}(\tau) = A(\tau)X(\tau), \quad \tau \in R^{\circ} \tag{1.9}$$

$$\dot{Y}(\tau) = B(\tau)Y(\tau), \quad \tau \in R^{\circ} \tag{1.10}$$

then it is easy to see that

$$T(\tau) = X(\tau)CY^{-1}(\tau), \ \tau \in R^{\circ} \tag{1.11}$$

with an arbitrary constant matrix C is the general solution of the matrix equation (1.8).

Indeed, taking into account

$$\frac{d}{d\tau}Y^{-1}(\tau) = -Y^{-1}(\tau)\frac{d}{d\tau}Y(\tau)Y^{-1}(\tau) = -Y^{-1}(\tau)B(\tau)$$

we verify that (1.11) satisfies equation (1.8).

Obviously, from $\det C \neq 0$ follows $\det T(\tau) \neq 0$, $\tau \in R^{\circ}$.

If we reverse the roles of equations (1.1) and (1.2), we arrive at the same result.

Conversely, under roles are solvable in equation (1.8) at $\tau \in R^{\circ}$ it is easy to verify that equations (1.1) and (1.2) are reducible to each other.

Thus, the following theorem is proved.

Theorem 1. For, under conditions (1.3) and (1.4), equations (1.1) and (1.2) are mutually reducible to each other at $\tau \in R^{\circ}$ it is necessary and sufficient that equation (1.8) is solvable at $\tau \in R^{\circ} \subset R$.

2. Mutual reducibility of linear equations in the class of continuous periodic functions

Before starting to investigate the question, we note that if instead of vector-matrix equations (1.1) and (1.2) we consider matrix equations

$$\dot{X} = A(\tau)X \,, \tag{2.1}$$

$$\dot{Y} = B(\tau)Y \,, \tag{2.2}$$

where X and Y are $n \times n$- matrices, $\dot{X}$ and $\dot{Y}$ are their derivatives by τ, then carrying out a similar study given in paragraph 1 would have obtained the result given in theorem 1.

In other words, the research of the issues of mutual reducibility can be carried out both in vector-matrix and in a purely matrix form.

Thus, following the last form, consider the system of matrix equations (2.1) and (2.2) with matrices $A(\tau)$ and $B(\tau)$, satisfying the conditions

$$A(\tau + \vartheta) = A(\tau) \in C(R) \,, \tag{2.3}$$

$$B(\tau + \vartheta) = B(\tau) \in C(R) \,. \tag{2.4}$$

Let us pose the problem of clarification conditions for the mutual reducibility of equations (2.1) and (2.2) under conditions (2.3) and (2.4) using a nonsingular linear substitution (1.6) with a transformation matrix $T(\tau)$, with property

$$\det T(\tau) \neq 0, \ \ \tau \in R: \ T(\tau + \theta) = T(\tau) \in C^{(1)}(R) \,. \tag{2.5}$$

To solve this problem, we introduce the matricants $U(\tau)$ and $V(\tau)$ of equations (2.1) and (2.2) into consideration:

$$\dot{U}(\tau) = A(\tau)U(\tau) \,, \ \ U(0) = E \,, \tag{2.6}$$

$$\dot{\mathrm{V}}(\tau) = B(\tau)V(\tau) \,, \ \ V(0) = E \,, \tag{2.7}$$

where E is the unit $n \times n$- matrix.

Further, according to Theorem 1, it is necessary to find a solution to equation (1.8) satisfying condition (2.5).

Obviously, equations (2.1) and (2.2) under conditions (2.3) and (2.4) are solvable for $\tau \in R$. Consequently, equation (1.8) is also solvable for $\tau \in R$, and according to (1.11)

$$T(\tau) = U(\tau)T_0 V^{-1}(\tau) \qquad (2.8)$$

where $C = T_0$ should be chosen so that the solution (2.8) satisfies condition (2.5).

It is known that matricants with θ-periodicity of the input data have the properties

$$U(\tau+\theta) = U(\tau)U(\theta), \qquad (2.9)$$

$$V(\tau+\theta) = V(\tau)V(\theta). \qquad (2.10)$$

Then, taking into account (2.9) and (2.10), from (2.8) we obtain

$$T(\tau+\theta) = U(\tau)U(\theta)T_0 V^{-1}(\theta)V^{-1}(\tau). \qquad (2.11)$$

Now, using the definition of θ - periodicity by virtue of (2.8) and (2.11), we have

$$U(\tau)T_0 V^{-1}(\tau) = U(\tau)U(\theta)T_0 V^{-1}(\theta)V^{-1}(\tau).$$

Hence, we obtain condition

$$U(\theta)T_0 = T_0 V(\theta) \qquad (2.12)$$

for matrix T_0, which ensures the θ - periodicity of matrix $T(\tau)$.

Conversely, from condition (2.12) we obtain the condition

$$T(\theta) = T(0) \qquad (2.13)$$

which ensures the θ - periodicity of the solution (2.8).

Therefore, the equivalence of conditions (2.12) and (2.13) guarantees the solvability of the problem posed. Since, according to [1-3], it is not always possible to indicate a matrix T_0, that satisfies equation (2.12), then from (2.12) it follows that the monodromy matrices of the equations under consideration are connected for mutual reducibility.

Thus proves the following theorem.

Theorem 2. In order for the matrix equations (2.1) and (2.2) with matrices (2.3) and (2.4) to be mutually reducible at means of the θ - periodic matrix $T(\tau) = T(\tau+\theta)$ it is necessary and sufficient that their monodromy matrices $U(\theta)$ and $V(\theta)$ are related to the relation (2.12) with a nonsingular matrix T_0.

If the condition of mutual reducibility is satisfied, then to determine the transformation matrix (2.8), instead of the matricants, one can use any fundamental matrices of equations. Indeed,

$$T(\tau) = U(\tau)X_0 X_0^{-1} T_0 Y_0 Y_0^{-1} V^{-1}(\tau) = X(\tau)T^0 Y^{-1}(\tau),$$

where $X(\tau) = U(\tau)X_0$, $Y(\tau) = V(\tau)Y_0$ are fundamental matrices, X_0, Y_0 are arbitrary nonsingular constant matrices, $T^0 = X_0^{-1} T_0 Y_0$ is a constant matrix.

3. Relationship between the mutual reducibility of periodic linear systems and Floquet's theorem.

To establish a connection between mutual reducibility, consider an equation with a periodic matrix of the form

$$\dot{U} = P(\tau)U \,, \tag{3.1}$$

and an equation with a constant matrix of the form

$$\dot{V} = KV \,, \tag{3.2}$$

where the matrices $P(\tau)$ and K satisfy the conditions

$$P(\tau + \vartheta) = P(\tau) \in C(R) \,, \tag{3.3}$$

$$K \in C_{const}(R) \,. \tag{3.4}$$

Here, $C_{const}(R)$ denotes the class constant for $\tau \in R$ of $n \times n$-matrices, and K is an unknown matrix, which should be chosen so that equation (3.1) is reducible to equation (3.2).

Since K also belongs to the class of θ-periodic matrices, then Theorem 2 is valid for equations (3.1) and (3.2), and

$$V(\tau) = e^{K\tau} \,, \tag{3.5}$$

and therefore, the corresponding monodromy matrix is determined by the relation

$$V(\theta) = e^{K\theta} \,. \tag{3.6}$$

Then, by virtue of (3.6), the condition of mutual reducibility, according to (2.12), has the for

$$U(\theta)T_0 = T_0 e^{-K\theta} \,. \tag{3.7}$$

Floquet's problem is to choose a constant matrix K in an appropriate way so that equation (3.1) is reducible to an equation of the form (3.2).

Then, to solve the Floquet problem on reducibility, it was always solved for equation (3.1) in condition (3.7) on mutual reducibility, both matrices K and matrices T_0 should be chosen.

Obviously, that it is always solvable if we put T_0 equal to the identity matrix E, and the matrix K is determined from (3.7) for $T_0 = E$. Therefore, from (3.7) we obtain

$$U(\theta) = e^{K\theta} \Leftrightarrow K = \frac{1}{\theta}\ln U(\theta) \tag{3.8}$$

and by virtue of (3.5) we have

$$U(\tau) = e^{\frac{\tau}{\theta}\ln U(\theta)} \,. \tag{3.9}$$

Since the matrix $T(\tau)$ is periodic with period θ, then from identity (2.8) we obtain a necessary and sufficient condition for the reducibility of equation (3.1) to equation (3.2) in the form

$$U(\tau) = T(\tau)e^{\frac{\tau}{\theta}\ln U(\theta)} \,. \tag{3.10}$$

Here, obviously, the $T(\tau)=\Phi(\tau)$ - periodic nonsingular Floquet matrix, $\Lambda=\frac{1}{\theta}\ln U(\theta)=K$ is constant matrix, and (3.10) represents the matricant of the periodic equation (3.1).

Hence, it is clear that the problem under consideration is broader than the Floquet problem associated with the reducibility of periodic systems indicated by Lyapunov.

Conversely, by virtue of the representation of the matricant $U(\tau)$ of the form (3.10) at $T_0=E$ condition (3.7) about on the mutual reducibility of equations (3.1) and (3.2) follows. Therefore, the representation of the matricant (3.10), which we write in the form

$$U(\tau)=\Phi(\tau)e^{\tau\Lambda} \tag{3.11}$$

is a necessary and sufficient condition for the reducibility of equation (3.1) to equation (3.2), where $\Phi(\tau)$ by virtue of (2.5) satisfies conditions

$$\det\Phi(\tau)\neq 0\,,\ \Phi(\tau+\theta)=\Phi(\tau)\,,\ \tau\in R\,, \tag{3.12}$$

and the matrix $\Lambda=K$ was determined by the relation (3.8).

Thus, the following theorem has been proved.

Theorem 3. For equation (3.1) with any periodic coefficients (3.3) to exist mutually reducible equation (3.2) with constant coefficients (3.4), it is necessary and sufficient that the matricant $U(\tau)$ of equation (3.1) be represented in the form (3.11) with matrices (3.8) and (3.12).

Theorem 3 proved can be called Floquet's theorem formulated in terms of the concept of mutual reducibility of differential equations.

4. Linear homogeneous systems mutually reducible in the sense of Lyapunov

In the theory of stability of linear systems of differential equations, the concept of the boundedness of a function $x(\tau)$ on the semiaxis $R_{\tau_0}=[\tau_0,+\infty)$ by the norm

$$\|x\|_{\tau_0}=\sup_{\tau\in R_{\tau_0}}|x(\tau)|<+\infty\,, \tag{4.1}$$

where $\tau_0\in R_0=[0,+\infty)$, $|.|$ is the sign of the vector norm of one of the known (cubic, octahedral, Euclidean) types.

Systems (1.1) and (1.2) with conditions (1.3) and (1.4) are mutually reducible using the Lyapunov matrix [1-3]:

$$\|L\|_{\tau_0}<+\infty\,,\ \|\dot{L}\|_{\tau_0}<+\infty\,,\ |\det L(\tau)|\geq 1=const>0 \tag{4.2}$$

are called systems mutually reducible in the sense of Lyapunov, where $\|L\|_{\tau_0}=\sup_{\tau\in R_{\tau_0}}|L(\tau)|$, $|.|$ is the sign of the matrix norm corresponding to the norm of the vector (4.1), $\dot{L}=\frac{d}{d\tau}L$.

Theorem 4.1. If one of the Lyapunov mutually reducible systems (1.1) and (1.2) with conditions (1.3) and (1.4) is stable, then the other is also stable.

Indeed, it is known that along with the matrix $L(\tau)$, its inverse matrix $L^{-1}(\tau)$ is also a Lyapunov matrix:

$$\|L^{-1}\|_{\tau_0} < +\infty, \ \left\|\frac{d}{d\tau}L^{-1}\right\|_{\tau_0} = \|L^{-1}\|_{\tau_0} < +\infty, \ |\det L^{-1}(\tau)| \geq l^* = const > 0 \tag{4.2*}$$

By the definition of mutual reducibility, we have

$$X(\tau) = L(\tau)Y(\tau) \tag{4.3}$$

for the matricants $X(\tau)$ and $Y(\tau)$ of systems (1.1) and (1.2) with properties (1.3) and (1.4).

From (4.3) we obtain

$$Y(\tau) = L^{-1}(\tau)X(\tau). \tag{4.3*}$$

Further, if we assume stable system (1.1) with condition (1.3), then we have

$$\|X\|_{\tau_0} < +\infty. \tag{4.4}$$

Then, by virtue of (4.2*), (4.3*) и (4.4) we obtain

$$\|Y\|_{\tau_0} < +\infty. \tag{4.5}$$

Obviously, the fulfillment of condition (4.5) is equivalent to the stability of system (1.2) with condition (1.4).

Conversely, if system (1.2) with condition (1.4) is stable, and hence condition (4.5) is satisfied, then, by virtue of (4.2), (4.3), and (4.5), we have condition (4.4).

This is equivalent to the Lyapunov stability of system (1.1) with condition (1.3).

Theorem 4.1 is completely proved.

Further, assume that the matrix $B(\tau)$ in system (1.2) is constant. Then we have the conditions

$$B = const \tag{4.6}$$

and the matrix $Y(\tau)$ is represented in the form

$$Y(\tau) = e^{B\tau}. \tag{4.7}$$

Substituting (4.7) with properties (4.6), (4.3), we obtain the N.P.Erugin's formula

$$X(\tau) = L(\tau)e^{B\tau} \tag{4.8}$$

on the reducibility of system (1.1) with condition (1.3) to the system

$$\frac{dy}{d\tau} = By \tag{4.9}$$

with constant matrix (4.6).

Note that, by Theorem 4.1, the stability of system (1.1) with condition (1.3) implies the stability of system (4.9). This means that all eigenvalues of matrix B have non-positive parts

$$\operatorname{Re}\lambda_j(B) \le 0, \ \ j = \overline{1,n},$$

moreover, only simple elementary divisors admit a zero eigenvalue.

Conversely, from the last properties of a constant matrix. B the condition follows

$$\sup_{\tau \in R_{\tau_0}} \left| \exp[B\tau] \right| < +\infty, \tag{4.10}$$

then, by virtue of (4.10) and (4.2), from (4.8) we have condition (4.4). Consequently, the stability of system (4.9) implies the Lyapunov stability of system (1.1) with condition (1.3).

From these arguments, we easily obtain the following theorem of Erugin in terms of mutual reducibility.

Теорема 4.2. For the mutual reducibility of systems (1.1) with conditions (1.3) and (4.9) with condition (4.6), it is necessary and sufficient that these systems be related to formula (4.8).

We will not dwell on the proof of this theorem, since it is well known. Note that in this case the systems can simultaneously be both stable and unstable.

In conclusion, we note that on the basis of [4-10], it is possible to obtain interesting results on the mutual reducibility of linear homogeneous systems of partial differential equations with multiperiodic coefficients, which are closely related to systems with almost periodic coefficients of ordinary differential equations.

References

1. Ляпунов А.М. Общая задача об устойчивости движении / А.М. Ляпунов. – М. Л.: ГИТТЛ, 1950. — 472 с.

2. Еругин Н.П. Приводимые системы / Н.П. Еругин // Тр. Матем. Ин-та им. В.А. Стеклова. — 1946. — Т. 13. — С. 3-96.

3. Демидович Б.П. Лекции по математической теории устойчивости / Б.П. Демидович. — М.: Наука, 1967. — 472 с.

4. Харасахал В.Х. Почти-периодическая решения обыкновенных дифференциальных уравнений / В.Х. Харасахал. — Алма-Ата: Наука, 1970. — 270 с.

5. Умбетжанов Д.У. Почти многопериодические решения дифференциальных уравнений в частных производных / Д.У. Умбетжанов. — Алма-Ата: Наука, 1979. — 210 с.

6. Мухамбетова А.А. Устойчивость решений систем дифференциальных уравнений с многомерным временем / А.А. Мухамбетова, Ж.А. Сартабанов. — Актобе: Принт А, 2007. — 168 с.

7. Кульжумиева А.А. Периодические решения систем дифференциальных уравнений с многомерным временем / А.А. Кульжумиева, Ж.А. Сартабанов. — Уральск: РИЦ ЗКГУ, 2013. — 167 с.

8. Сартабанов Ж.А. О приводимости линейных систем дифференциальных уравнений с многоперидическими коэффициентами / Ж.А. Сартабанов // Изв. АК КазССР. Сер. физ.-мат., — 1989, №5. — С. 34-40.

9. Сартабанов Ж.А. К линейных систем дифференциальных уравнений с квазипериодическими коэффициентами / Ж.А. Сартабанов. — Дсп. в КазГос ИНТИ. 27.03.1995. Р. 5967-Ка95. — 15 с.

10. Сартабанов Ж.А. Приводимость линейных многопериодических уравнений с оператором дифференцирования по диагонали / Ж.А. Сартабанов, А.А. Кульжумиева // Математический журнал МОН РК. —2018. — Т. 18, №1. —С. 139 -150.

References

1. Lyapunov A.M. (1950). Obshaya zadacha ob ustoichivosti dvizhenii [General problem of mation stability]. M.-GITTL [in Russian].

2. Erugin N.P. (1946). Privodimye sistemy [Reduced systems], Tr. Matem. In-ta im. V.A. Steklova, Vol. 13. 3-96 [in Russian].

3. Demidovich B.P. (1970). Lekcii po matematicheskoi teorii ustoichivosti [Lectures on the mathematical theory of stability]. Alma-Ata: Nauka [in Russian].

4. Kharasakhal V.Kh. (1970). Pochti-periodicheskaya resheniya obyknovennyh differencialnyh uravnenii [Almost-periodic solution of ordinary differential equations]. Alma-Ata: Nauka [in Russian].

5. Umbetzhanov D.U. (1979). Pochti mnogoperiodicheskie resheniya differencialnyh uravnenii v chastnyh proizvadnyh [Almost multiperiodic solutions to partial differential equations]. Alma-Ata: Nauka [in Russian].

6. Mukhambetova A.A., Sartabanov Zh.A. (2007). Ustoichvost reshenii system differencialnyh uravnenii s mnogomernym vremenem [Stability solutions of systems of differential equations with multidimensional time]. Aktobe: Print A [in Russian].

7. Kulzhumiyeva A.A., Sartabanov Zh.A. (2013). Periodicheskie resheniya sistem differencialnyh uravnenii s mnogomernym vremenem [Periodic solutions of systems of differential equations with multidimensional time]. Uralsk: RITs ZKGU [in Russian].

8. Sartabanov Zh.A. (1989). O privodimosti linainyh sistem differencialnyh uravnenii s mnogoperiodicheskimi koefficientami [On the reducibility of linear systems of differential equations with multiperiodic coefficients]. Izv. AK KazSSR. Ser. phys.-mat., № 5, 34-40 [in Russian].

9. Sartabanov Zh.A. (1995). K linainyh sistem differencialnyh uravnenii s kvaziperiodicheskimi koefficientami [On linear systems of differential equations with quasiperiodic coefficients]. Dsp v KazGos INTI. R. 5967-Ka95 [in Russian].

10. Sartabanov Zh.A., Kulzhumiyeva A.A. (2018). Privodimost lineinyh mnogoperiodicheskih uravnenii s operatorom differencirovaniya po diagonali [Reducibility of linear multiperiodic equations with diagonal differentiation operator]. Mathematicheskii zhurnal — Mathematical journal, №1, 139-150 [in Russian].

СЫЗЫҚТЫ ӨЗАРА КЕЛТІРІМДІ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР

Ж.А. САРТАБАНОВ[1,*], Г.М. АЙТЕНОВА[2], Г.С. ТӨРЕМҰРАТОВА[1]

[1]Қ. Жұбанов атындағы Ақтөбе өңірлік университеті, Ақтөбе, Қазақстан

[2]М.Өтемісов атындағы Батыс Қазақстан университеті, Орал, Қазақстан

[*]e-mail: sartabanov42@mail.ru

Аңдатпа. Мақала Ляпуновтың класикалық жұмысынан [1] өз бастауын алатын қарапайым дифференциалдық теңдеулер жүйесінің келтірімділігіне арналады. Н.П. Еругин [2] жұмысында бұл зерттеудің идеясын дамытты. Келтірімділік теориясының тартымды бағыты тербелмелі коэффициентті сызықты жүйелердің келтірімділік мәселесі болып табылады, периодты жағдайда белгілі Флоке [3] теориясын аламыз.

Айнымалы коэффициентті сызықты жүйелердің тұрақты коэффициентті жүйелерге келтірімділігі жөніндегі сұрақ ерекше қызығушылық тудыратындығын ескерген жөн.

Бұл зерттеуде осы сұрақ жалпылама тұрғыдан қойылған. Егер екі айнымалы каэффициентті сызықты жүйелер ерекше емес сызықты ауыстыру негізінде бір-біріне келтірілімді болса, яғни өзара келтірілімді болса, онда осы жүйелер мен олардың түрлендірулерінің арасында болатын байланысты анықтау жөнінде есеп туындайды. Бұл мақаланың мақсаты осы есепті кейбір дербес жағдайларда зерттеуге және қарапайым дифференциалдық теңдеулер жүйесінің келтірімділігі бойынша белгілі нәтижелермен байланысты орнатуға негізделеді.

Берілген есепті зерттеу барысында есептің қойылымы келтіріледі; осы есептің шешілімділігі жөнінде сұрақ қарастырылады; периодты коэффициентті сызықты жүйелердің өзара келтірімділігі зерттеледі; өзара келтірімділік сұрағы мен Флоке теориясы арасында байланыс орнатылады және қозғалыстың орнықтылығы теориясынан белгілі Ляпунов матрицасы арқылы жүйелердің өзара келтірімділік сұрағы зерттеледі.

Қорытындыда бұл жұмыста алынған нәтижелер [4-10] жұмыстарында қарастырылған дербес туындылы сызықты теңдеулер жүйесі жағдайында жалпылануы мүмкін екендігі ескеріледі.

Түйін сөздер: сызықты жүйе; өзара келтірімділік; матрицант; түрлендіру матрицасы; периодты жүйе; Флоке теоремасы; Ляпунов матрицасы; орнықтылық.

ЛИНЕЙНЫЕ ВЗАИМНО ПРИВОДИМЫЕ ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ

Ж.А. САРТАБАНОВ[1*], Г.М. АЙТЕНОВА[2], Г.С. ТОРЕМУРАТОВА[1]

[1]Актюбинский региональный университет им. К.Жубанова, Актобе, Казахстан

[2]Западно-Казахстанский университет имени М. Утемисова, Уральск, Казахстан

[*]e-mail: sartabanov42@mail.ru

Аннотация. Статья посвящена вопросу приводимости систем обыкновенных дифференциальных уравнений, который берет свое начало с классической работы Ляпунова [1]. Идея этого исследования была развита Еругином Н.П. в работе [2]. Привлекательным направлением теории приводимости являются проблемы приводимости линейных систем с колебательными коэффициентами, в частности, в периодическом случае имеем известную теорию Флоке [3].

Следует отметить, что особый интерес представляет вопрос приводимости линейных систем с переменными коэффициентами к системам с постоянными коэффициентами.

В данном исследовании этот вопрос поставлен с общей точки зрения. Если две линейные системы с переменными коэффициентами на основе неособенной линейной заменой приводимые друг к другу, то есть взаимно приводимые, то возникает задача о выяснении связи, существующей между этими системами и их преобразованиями. Цель этой статьи заключается в изучении этой задачи в некоторых частных случаях и в установлении связи с известными результатами по приводимости систем обыкновенных дифференциальных уравнений.

В связи с решением данной задачи в заметке приводится постановка вопроса; рассматривается вопрос о разрешимости этой задачи; исследуется взаимная приводимость линейных систем с периодическими коэффициентами; устанавливается связь между вопросом взаимной приводимости и теории Флоке и изучается вопрос взаимной приводимости систем при помощи матрицы Ляпунова, известной из теории устойчивости движений.

В заключении отмечается, что полученные результаты в данной работе могут быть обобщены в случае линейных систем уравнений в частных производных рассмотренных в работах [4-10].

Ключевые слова: линейная система; взаимная приводимость; матрицант; матрица преобразования; периодичная система; теорема Флоке; матрица Ляпунова; устойчивость.