## ФИЗИКА-МАТЕМАТИКА ҒЫЛЫМДАРЫ
## ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ
## PHYSICAL AND MATHEMATICAL SCIENCES

**IRSTI 28.23.37**

# CHAOTIC OSCILLATIONS IN NEURAL NETWORKS

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**Abstract.** Neural networks, also known as artificial neural networks have special significance in adaptive pattern recognition, vision, image processing, associative memory, enhancement of X-Ray and computed tomography images. Similar to the brain, neural networks are built up of many neurons with many connections between them. In this paper, a new type of oscillation, unpredictable, for the neural networks such as Hopfield-type neural networks (HNNs), shunting inhibitory cellular neural networks (SICNNs) and inertial neural networks (INNs) is proposed. Unpredictable oscillations are a completely new type of motion considered in the field of neuroscience. For each neural network model, the existence and exponential stability of a unique strongly unpredictable oscillation are investigated. The presence of chaotic motion in neural network is approved by existence of unpredictable solutions. For the first time in the literature, Hopfield-type neural networks, shunting inhibitory cellular neural networks and inertial neural networks with unpredictable perturbations were considered. In this article, we summarize the main results of the study of unpredictable oscillations of neural networks. Additionally, to the theoretical analysis, we have provided numerical simulation, considering that all of the assumed conditions are fulfilled.

**Key words:** Hopfield-type neural networks, Shunting inhibitory cellular neural networks, Inertial neural networks, Unpredictable oscillations, Strongly unpredictable oscillations, Poincaré chaos, Asymptotic stability.

Neural networks in present research display unpredictable oscillations and chaos. The unpredictable function was introduced in [1] and is based on the dynamics of unpredictable points and Poincaré chaos [2]. More precisely, the function is an unpredictable point of the Bebutov dynamics, and consequently, it is a member of the chaotic set [3]. The notion of the unpredictable point extends the frontiers of the classical theory of dynamical systems, and the unpredictable function provides new problems of the existence of unpredictable oscillations for the theory of differential equations [1-9].

The HNNs [10-12], SICNNs [13-15] and INNs [16-18] are artificial neural networks can be used in adaptive pattern recognition, vision, image processing, associative memory, enhancement of

X-Ray images and medical image restoration. They play an important role in medicine, computer science, robotics, biophysics and psychophysics.

Let $N$ and $R$ be the set of natural and real numbers, respectively. Additionally, introduce the norm $\|v\| = \max_i |v_i|, i = 1, \dots, p,$ where $|\cdot|$ - is the absolute value, and $v_i \in R, i = 1, \dots, p$. And $\|A\| = \max_i \sum_{j=1}^{p} |a_{ij}|, \quad i = 1, \dots, p, \ j = 1, \dots, p,$ means the norm for the $p \times p$ matrix $A = \{a_{ij}\}$.

The following definitions are the main in our research.

**Definition 1.** [1] *A uniformly continuous and bounded function $\vartheta: R \to R^p$ is unpredictable if there exist positive numbers $\varepsilon_0, \delta$ and sequences $\{t_n\}, \{s_n\}$ both of which diverge to infinity such that $\|\vartheta(t + t_n) - \vartheta(t)\| \to 0$ as $n \to \infty$ uniformly on compact subsets of $R$ and $\|\vartheta(t + t_n) - \vartheta(t)\| \geq \varepsilon_0$ for each $t \in [s_n - \delta, s_n + \delta]$ and $n \in N$.*

**Definition 2.** [7] *A uniformly continuous and bounded function $v: R \to R^p, v = (v_1, v_2, \dots, v_p)$, is strongly unpredictable if there exist positive numbers $\varepsilon_0, \delta$ and sequences $\{t_n\}, \{s_n\}$, both of which diverge to infinity such that $v(t + t_n) \to v(t)$ as $n \to \infty$ uniformly on compact sets of $R$ and $|v_i(t + t_n) - v_i(t)| \geq \varepsilon_0$ for all $i = 1,2\dots, p$, $t \in [s_n - \delta, s_n + \delta]$, and $n \in N$.*

**Main results.**

Let us consider the following HNNs,

$$x_i{}'(t) = -a_i x_i(t) + \sum_{j=1}^{p} b_{ij} f_j(x_j(t)) + v_i(t) \tag{1}$$

where $t \in R, x_i \in R, \ i = 1, \dots, p$:

$p$- the number of neurons in the network;

$x_i(t) -$ the membrane potential of the unit $i$, at time $t$;

$a_i > 0 -$ the rates with which the units self-regulate or reset their potentials when isolated from other units and inputs;

$f_j -$ the measures of activation to its incoming potentials of the unit $i$, at time $t$;

$b_{ij} -$ the synaptic connection weight of the unit $j$ on the unit $i$;

$v_i -$ the external input from outside the network to the unit $i$,

Suppose that the coefficients $b_{ij} \in R$, the activation functions $f_j, v_i: R \to R$ are continuous functions.

Denote by $\mathcal{P}$ the space of vector-functions, $\varphi: R \to R^p$, $\varphi = (\varphi_1, \varphi_2, \dots, \varphi_p)$, such that $\|\varphi\| = \max_{t \in R} \|\varphi(t)\|$, satisfying the following conditions:

**($\mathcal{P}$1)** $\varphi(t)$ are uniformly continuous;

**($\mathcal{P}$2)** $\|\varphi(t)\|_1 < H$ for all $\varphi(t) \in \mathcal{P}$, where $H$ is positive number;

**($\mathcal{P}$3)** there exists a sequence $t_n, t_n \to \infty$ as $n \to \infty$ such that for each $\varphi(t) \in \mathcal{P}$ the sequence $\varphi(t+t_n)$ uniformly converges to $\varphi(t)$ on each closed and bounded interval of the real axis.

The next conditions are required:

(**$C1$**) the $v(t) = (v_1(t), v_2(t), \dots, v_p(t))$, $t, v_i(t) \in R$ in system (1) belongs to space $\mathcal{P}$ and is strongly unpredictable with positive numbers $\delta, \varepsilon_0$ and sequences $s_n$ as $n \to \infty$, of positive integers, which satisfy $\|\vartheta(t+t_n) - \vartheta(t)\| \geq \varepsilon_0$ for each $t \in [s_n - \delta, s_n + \delta]$ and $n \in N$.

(**$C2$**) there exists a positive number $L$ such that $|f_i(u) - f_i(v)| \leq L|u - v|$ for all $u, v \in R$;

(**$C3$**) the inequalities $1 < \gamma \leq a_i \leq \bar{\gamma}$, $i = 1, \dots, p$, are valid with positive numbers $\gamma$ and $\bar{\gamma}$;

(**$C4$**) $|v_i(t)| < H$ and $|f_i(t)| < \overline{m}_i$ where $\overline{m}_i > 0$, $i = 1, \dots, p$ and $t \in R$;

(**$C5$**) $\frac{\max_i \sum_{j=1}^{p} |b_{ij}| \, \overline{m}_j}{\gamma - 1} < H$;

(**$C6$**) $L \max_i \sum_{j=1}^{p} |b_{ij}| < \gamma$.

The next theorem is true.

**Theorem 1.** *Assume that the function $v = (v_1, v_2, \dots, v_p)$ in neural network (1) is strongly unpredictable and conditions (C2) -(C6) are valid. Then, the neural network (1) possesses a unique asymptotically stable strongly unpredictable solution.*

Next, consider the following SICNNs,

$$\frac{dx_{ij}}{dt} = -b_{ij}x_{ij} - \sum_{D_{kp} \in N_r(i,j)} D_{ij}^{kp} f(x_{kp}(t)) x_{ij}(t) + g_{ij}(t), \tag{2}$$

with strongly unpredictable perturbations.

Denote by $\mathcal{B}$ the set of functions $u(t) = (u_{11}, \dots, u_{1n}, \dots, u_{m1} \dots, u_{mn})$, $t, u_{ij} \in R$, $i = 1,2, \dots, m, j = 1,2, \dots, n$, where $m, n \in N$, such that:

**($\mathcal{B}$1)** $u(t)$ are uniformly continuous and bounded functions, $||u||_1 < H$ for all $u(t) \in \mathcal{B}$, where $H > 0$;

**($\mathcal{B}$2)** there exists a sequence $t_p, t_p \to \infty$ as $p \to \infty$ such that for each $u(t) \in \mathcal{B}$ the sequence $u(t + t_p)$ uniformly converges to $u(t)$ on each closed and bounded interval of the real.

As well as the next conditions are needed:

(**$D1$**) the functions $g_{ij}(t) \in \mathcal{B}$, $i = 1,2, \dots, m, j = 1,2, \dots, n$, and they are unpredictable with numbers $\delta, \varepsilon_0 > 0$ and a sequence $t_p \to \infty$ as $p \to \infty$, which satisfy $|g_{ij}(t + t_p) - g_{ij}(t)| \geq \varepsilon_0$ for all $t \in [s_p - \delta, s_p + \delta]$.

(**$D2$**) $\gamma \leq b_{ij} \leq \bar{\gamma}$, where $\gamma, \bar{\gamma}$ are positive numbers;

(**$D3$**) $|g_{ij}(t)| \leq m_{ij}$, where $m_{ij} > 0$, $t \in R$;

(**$D4$**) $|f(s)| \leq m_f$, where $m_f > 0$ for $|s| < H$;

(**$D5$**) there exists positive number $L$ such that $|f(s_1) - f(s_2)| \leq L|s_1 - s_2|$ for all $s_1, s_2$,

$|s_1| < H, |s_2| < H$;

$(\boldsymbol{D6}) m_f \sum_{D_{kp}\in N_r(i,j)} D_{ij}^{kp} < b_{ij}$;

$(\boldsymbol{D7}) \frac{m_{ij}}{b_{ij} - m_f \sum_{D_{kp}\in N_r(i,j)} D_{ij}^{kp}} < H$;

$(\boldsymbol{D8})\ (LH + m_f) \max_{(i,j)} \sum_{D_{kp}\in N_r(i,j)} D_{ij}^{kp} < \gamma$.

The following theorem is proved.

**Theorem 2.** *If (D1) -(D8) are valid, then the neural network (2) admits a unique asymptotically stable strongly unpredictable solution.*

Finally, the following INN is considered:

$$\frac{d^2x_i(t)}{dt^2} = -a_i\frac{dx_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{p} c_{ij}f_j(x_j(t)) + v_i(t), \tag{3}$$

where $t, x_i \in R, i = 1,2,\dots,p$. The coefficients $c_{ij} \in R$, the activation functions $f_i: R \to R$ such that

(**I1**) $|f_i(x_1) - f_i(x_2)| \le L_i|x_1 - x_2|$, $L_i > 0$, for all $i = 1,2,\dots,p,\ x_1, x_2 \in R$, and $L = \max_{1\le i\le p} L_i$.

By transformation

$$y_i(t) = \xi_i\frac{dx_i(t)}{dt} + \zeta_ix_i(t), i = 1,\dots,p, \tag{4}$$

he system (3) can be rewritten as

$$\frac{dx_i(t)}{dt} = -\frac{\zeta_i}{\xi_i}x_i(t) + \frac{1}{\xi_i}y_i(t), \tag{5}$$

$$\frac{dy_i(t)}{dt} = -\left(a_i - \frac{\zeta_i}{\xi_i}\right)y_i(t) - \left(\xi_ib_i - \zeta_i\left(a_i - \frac{\zeta_i}{\xi_i}\right)\right)x_i(t) +$$

$$+\xi_i\sum_{j=1}^{p} c_{ij}f_j(x_j(t)) + \xi_iv_i(t), \tag{6}$$

The following conditions are needed:

(**I2**) the functions $v_i(t)$, are unpredictable;

(**I3**) there exists a positive number $H$ and $M_f$ such that $|f_i(s)| \le M_f, i = 1,\dots,p, |s| < H$.

Moreover, assume that for positive real numbers $\zeta_i$ and $\xi_i,\ i = 1,\dots,p$ the following assumptions are valid:

(**I4**) $a_i > \frac{\zeta_i}{\xi_i} + \xi_i, \zeta_i > \xi_i > 1$;

(**I5**) $(a_i - \frac{\zeta_i}{\xi_i}) - (|\zeta_i\left(a_i - \frac{\zeta_i}{\xi_i}\right) - \xi_ib_i| + \xi_i) > 0$;

**(I6)** $\dfrac{\xi_i M_f \sum_{j=1}^{p} c_{ij}}{(a_i - \frac{\zeta_i}{\xi_i}) - (|\zeta_i\left(a_i - \frac{\zeta_i}{\xi_i}\right) - \xi_i b_i| + \xi_i)} < H;$

**(I7)** $\dfrac{1}{(a_i - \frac{\zeta_i}{\xi_i})}(|\zeta_i\left(a_i - \frac{\zeta_i}{\xi_i}\right) - \xi_i b_i| + L\xi_i \sum_{j=1}^{p} c_{ij}) < 1;$

**(I8)** $\max_i(\frac{1}{\xi_i}, |\zeta_i\left(a_i - \frac{\zeta_i}{\xi_i}\right) - \xi_i b_i| + L\xi_i \sum_{j=1}^{p} c_{ij}) < \min_i(\frac{\zeta_i}{\xi_i}, a_i - \frac{\zeta_i}{\xi_i}).$

The main theorem is proved.

**Theorem 3.** *The neural network (3) admits a unique exponentially stable unpredictable solution if* $(I1) - (I8)$ *are valid.*

**Example 1.** Let us introduce the following Hopfield type neural network,

$$x_i'(t) = -a_i x_i(t) + \sum_{j=1}^{3} b_{ij} f_j(x_j(t)) + v_i(t) \quad (7)$$

where $i = 1, 2, 3$ and $a_1 = 3, a_2 = 4,\ a_1 = 5.\ f(x(t)) = \frac{1}{2} arctg(x(t))$.

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 0.02 & 0.03 & 0.02 \\ 0.04 & 0.05 & 0.01 \\ 0.03 & 0.06 & 0.02 \end{pmatrix},$$

and $v_1(t) = 60\Theta^3(t) + 4$ , $v_2(t) = -70\Theta^3(t) + 3$ , $v_3(t) = 9\Theta(t) - 2.7$ , where $\Theta(t) = \int_{-\infty}^{t} e^{-3(t-s)}\Omega(s)ds$ is the unpredictable function [9]. The conditions $(C1) - (C6)$ are satisfied for the network (7) with $\gamma = 3,\ L = 0{,}5, H = 2.1$. The numerical results for the neural network (7) with the initial values $\phi_1(0) = 0.593,\ \phi_2(0) = 0.131,\ \phi_3(0) = 0.345,$ are shown in Figure 1. The solution $\phi(t)$ asymptotically converges to the unpredictable solution $x(t)$ as $t$ increases.

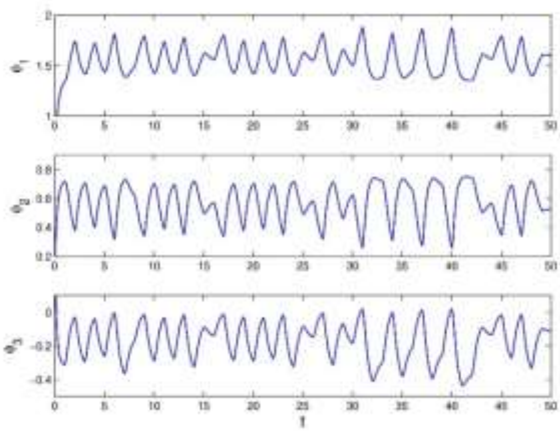

Figure 1 – **The time series of the coordinates $\phi_1,\ \phi_2,\ \phi_3$, of neural network (7)**

**Example 2.** Consider the following SICNNs:

$$\frac{dx_{ij}}{dt} = -b_{ij}x_{ij} - \sum_{D_{kp}\in N_1(i,j)} D_{ij}^{kp} f\left(x_{kp}(t)\right) x_{ij}(t) + g_{ij}(t), \tag{8}$$

where $i, j = 1,2,3,$

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 3 & 8 & 5 \\ 5 & 7 & 6 \\ 2 & 9 & 3 \end{pmatrix}, \quad \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} = \begin{pmatrix} 0.04 & 0.02 & 0.03 \\ 0 & 0.06 & 0.02 \\ 0.05 & 0 & 0.01 \end{pmatrix},$$

and $f(s) = 0.05arctg(s)$, $g_{11}(t) = 25\Theta^3(t) + 1$, $g_{12}(t) = 4\Theta(t)$, $g_{13}(t) = -3\Theta(t) + 2$, $g_{21}(t) = 2\Theta(t) + 2$, $g_{22}(t) = 17\Theta^3(t)$, $g_{23}(t) = 19\Theta(t) - 1$, $g_{31}(t) = -7\Theta(t) + 2$, $g_{32}(t) = 3\Theta(t)$, $g_{33}(t) = -13\Theta^3(t) + 2$, where $\Theta(t) = \int_{-\infty}^{t} e^{-3(t-s)}\Omega(s)ds$ is the unpredictable function. Moreover, according to the properties of unpredictable functions, the functions $g_{ij}(t), i = 1,2,3, j = 1,2,3$, are unpredictable. We have that $|g_{ij}(t)| \le m_{ij}$, where $m_{11} = 1.93$; $m_{12} = 1.34$; $m_{13} = 3$; $m_{21} = 8$; $m_{22} = 2.67$; $m_{23} = 7.34$; $m_{31} = 4.34$; $m_{32} = 1$; $m_{33} = 2.49$. One can calculate that $\sum_{D_{kp}\in N_1(1,1)} D_{11}^{kp} = 0.20$, $\sum_{D_{kp}\in N_1(1,2)} D_{12}^{kp} = 0.22$, $\sum_{D_{kp}\in N_1(1,3)} D_{13}^{kp} = 0.11$, $\sum_{D_{kp}\in N_1(2,1)} D_{21}^{kp} = 0.25$, $\sum_{D_{kp}\in N_1(2,2)} D_{22}^{kp} = 0.35$, $\sum_{D_{kp}\in N_1(2,3)} D_{23}^{kp} = 0.19$, $\sum_{D_{kp}\in N_1(3,1)} D_{31}^{kp} = 0.18$, $\sum_{D_{kp}\in N_1(3,2)} D_{32}^{kp} = 0.28$, $\sum_{D_{kp}\in N_1(3,3)} D_{33}^{kp} = 0.17$. The assumptions ($\mathcal{B}1$) -($\mathcal{B}8$) hold for the equation (8) with $\gamma = 2$, $\bar{\gamma} = 9$, $m_f = 0.08$, $H = 2.1$ and $L = 0.06$.

In Figure 2 shown the coordinates of $\phi(t)$ with $\phi_{11}(0) = 1.0245$, $\phi_{12}(0) = 0.2996$, $\phi_{13}(0) = 0.0837$, $\phi_{21}(0) = 0.8283$, $\phi_{22}(0) = 0.0413$, $\phi_{23}(0) = 1.8122$, $\phi_{31}(0) = 1.0678$, $\phi_{32}(0) = 0.2013$, $\phi_{33}(0) = 0.1$. The graphs confirm one more time the presence of chaos in the dynamics of the network (8).

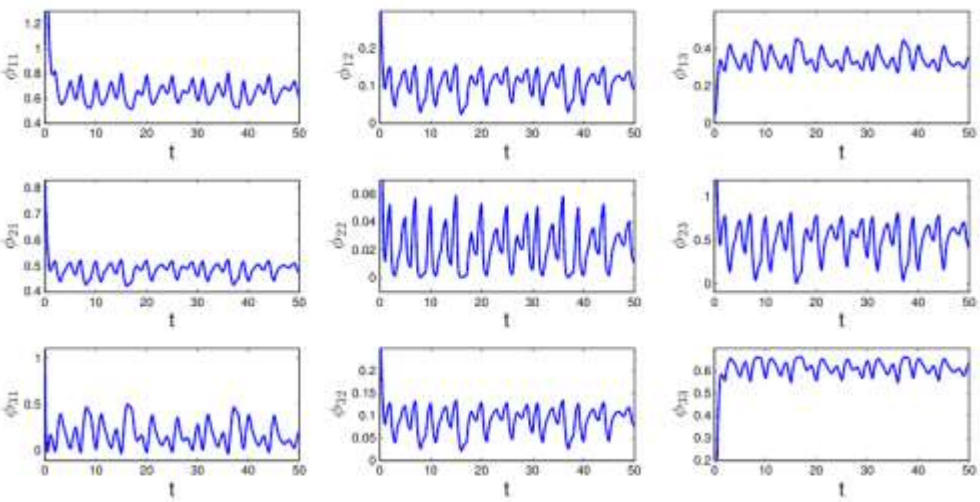

Figure 2 - **The coordinates of $\phi(t)$ asymptotically converge to the coordinates of the strongly unpredictable solution $x(t)$ of the network (8)**

**Example 3.** Let us take into account the system,

$$\frac{d^2x_i(t)}{dt^2} = -a_i\frac{x_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{3} c_{ij}f_j(x_j(t)) + v_i(t), \qquad (9)$$

$i = 1,2,3,\ a_1 = 6, a_2 = 7, a_3 = 5, b_1 = 8, b_2 = 6, b_3 = 8,\ f(x) = 0.36arctg(x)$,

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} = \begin{pmatrix} 0.02 & 0.03 & 0.02 \\ 0.04 & 0.05 & 0.02 \\ 0.04 & 0.06 & 0.02 \end{pmatrix}$$

and $v_1(t) = -58\Theta^3(t) + 5$, $v_2(t) = 76\Theta^3(t) + 4, v_3(t) = 42\Theta(t) - 3$, $\Theta(t) = \int_{-\infty}^{t} e^{-3(t-s)}\Omega(s)ds$. The function $v(t)$ is unpredictable in accordance with properties of unpredictable functions. The conditions $(I1)\ -(I8)$ hold for the network (9) with $\xi_1 = \xi_2 = 2, \xi_3 = 3, \zeta_1 = \zeta_2 = 4, \zeta_3 = 4.4$, $L = 0.36, M_f = 0.56, H = 17.1$. The simulation results for system (9) corresponding to the initial value $\omega_1(0) = 1.123, \omega_2(0) = 1.626, \omega_3 = 0.275$ are shown in Figure 3. The function $\omega(t)$ approximates the coordinates of the unpredictable solution $x(t)$ of the equation (9), as time increases. The figure reveals the irregular behavior of system (9).

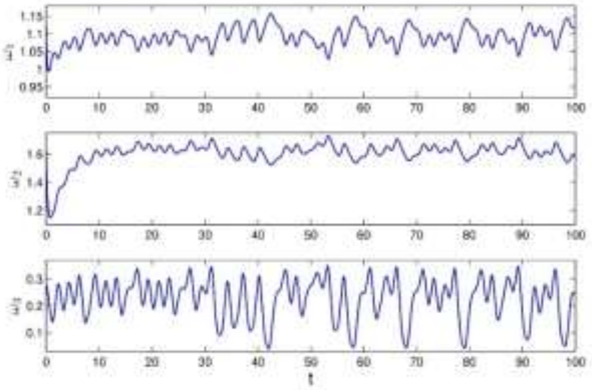


Figure 3 - **The irregular behavior of the coordinates of $\omega(t)$, of the network (9)**

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## НЕЙРОНДЫҚ ЖЕЛІЛЕРДЕГІ ХАОСТЫҚ ТЕРБЕЛІСТЕР

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**Аңдатпа.** Бізге жасанды нейрондық желілер атауымен де таныс болған нейрондық желілер, бейімделгіш үлгіні тану мен көруде, суретті өңдеуде, ассоциативті жады мен рентген және компьютерлік томографиялық кескіндерді жақсартуда ерекше маңызға ие. Нейронды желілер, ми құрлымына ұқсас, өзара байланысқан көптеген нейрондардан тұрады. Бұл жұмыста нейрондық желілер үшін болжанбайтын тербелістердің жаңа түрі ұсынылатын болады, мысалы, Хопфилд типті нейрондық желілер (ХНЖ), шунтаушы тежегіші бар жасушалық нейрондық желілер (ШТЖНЖ) және инерциялық нейрондық желілер (ИНЖ). Болжанбайтын тербелістер - бұл нейроғылым саласында кездесетін қозғалыстың мүлдем жаңа түрі болып табылады. Нейрондық желінің әр моделі үшін болжанбайтын тербелістің бар болуы және экспоненциалды орнықтылығы зерттеледі. Нейрондық желіде хаостық қозғалыстың болуы болжанбайтын шешімдердің бар болуымен расталады. Әдебиетте алғаш рет болжанбайтын қозулары бар Хопфилд типті нейрондық желілер, шунтаушы тежегіші бар жасушалық нейрондық желілер және инерциялық нейрондық желілер қарастырылды. Бұл мақалада, біз нейрондық желілердің болжанбайтын тербелістерін зерттеудің негізгі нәтижелерін қорытындылайтын боламыз. Теориялық талдаудан басқа, біз барлық болжанған шарттардың орындалғанын ескере отырып, сандық модельдеуді жүргіздік.

**Түйін сөздер:** Хопфилд типті нейрондық желілер, Шунтаушы тежегіші бар жасушалық нейрондық желілер, Инерциялық жүйке желілері, Болжанбайтын тербелістер, Қатты болжанбайтын тербелістер, Пуанкаре хаосы, Асимптотикалық орнықтылық.

## ХАОТИЧЕСКИЕ КОЛЕБАНИЯ В НЕЙРОННЫХ СЕТЯХ

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**Аннотация.** Нейронные сети, также известные как искусственные нейронные сети, имеют особое значение в адаптивном распознавании образов, зрении, обработке изображений, ассоциативной памяти, улучшении рентгеновских и компьютерных томографических изображений. Подобно мозгу, нейронные сети состоят из множества нейронов со множеством связей между ними. В данной работе предложен новый тип колебаний, непредсказуемый для таких нейронных сетей, как нейронные сети типа Хопфилда (НСХ), клеточные нейронные сети с шунтирующим торможением (КНСШТ) и инерционные нейронные сети (ИНС). Непредсказуемые колебания — это совершенно новый тип движения, рассматриваемый в области нейронауки. Для каждой модели нейронной сети исследуется существование и экспоненциальная устойчивость единственного сильно непредсказуемого колебания. Наличие хаотического движения в нейронной сети подтверждается наличием непредсказуемых решений. Впервые в литературе были рассмотрены нейронные сети типа Хопфилда, клеточные нейронные сети с шунтирующим торможением и инерционные нейронные сети с непредсказуемыми возмущениями. В данной статье мы обобщаем основные результаты исследования непредсказуемых колебаний нейронных сетей. Дополнительно к теоретическому анализу мы провели численное моделирование, учитывая, что все предполагаемые условия выполнены.

**Ключевые слова:** Нейронные сети типа Хопфилда, Клеточные нейронные сети с шунтирующим торможением, Инерционные нейронные сети, Непредсказуемые колебания, Сильно непредсказуемые колебания, Хаос Пуанкаре, Асимптотическая устойчивость.