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ON THE COMPACTNESS OF THE RIESZ POTENTIAL IN MORRY-TYPE SPACES

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Abstract. In this paper, we study the compactness of the generalized truncated Riesz potential on various versions of Morrey-type spaces. The main attention is paid to obtaining necessary and sufficient conditions that ensure the compactness of this operator. For this purpose, a detailed analysis of the functional properties of the generalized truncated Riesz potential is carried out, which made it possible to identify exact conditions on the space parameters under which the operator is compact. In particular, theorems are proved that establish a relationship between the parameters of Morrey-type spaces and the characteristics of the operator, which made it possible to expand the existing results previously known only in special cases. The conditions considered in the paper are not only sufficient, but also turn out to be necessary, which emphasizes their accuracy and optimality. The results obtained play an important role in the theory of integral-type operators and can also be applied in other areas of analysis, such as the theory of differential equations and theoretical physics. Thus, the study makes a significant contribution to the study of Riesz potential operators in Morrey-type spaces and opens up new prospects for future studies in this field. A similar result for the generalized truncated Riesz potential in L_p spaces was obtained in [10].

Key words: Morrey spaces, pre-compactness, Riesz potentials, totally bounded, local Morrey spaces.

First, let us introduce some definitions.

Definition 1. Let $0 < p, \theta \le \infty$, and let w be non-negative, measurable functions on $(0, \infty)$. We denote by $LM_{p\theta,w} \equiv LM_{p\theta,w}(\mathbb{R}^n)$ the local Morrey-type space[1]. This is the space of all functions $g \in L_p^{\mathrm{loc}}(\mathbb{R}^n)$ with finite quasi-norm:

$$\|g\|_{LM_{p\theta,w}} = \left\| w(r) \|g\|_{L_p(B(0,r))} \right\|_{L_\theta(0,\infty)} = \left(\int_0^\infty \left| w(r) \left(\int_{B(0,r)} |g(y)|^p \, dy \right)^{\frac{1}{p}} \right|^\theta dr \right)^{\frac{1}{\theta}}$$

where $B(0, r)$ is the ball centered at 0 with radius r.

The local Morrey-type space $LM_{p\theta,w}$ coincides with the space $LM_{p\theta}^{\lambda}$ when $w(r) = r^{-\lambda}$ and the quasi-norm is given by

$$\| g \|_{LM_{p\theta}^{\lambda}} \equiv \| g \|_{LM_{p\theta}^{\lambda}(\mathbb{R}^n)} = \left(\int_0^\infty \left(\frac{\|g\|_{L_p(B(0,r))}}{r^\lambda} \right)^\theta \frac{dr}{r} \right)^{1/\theta}$$

Definition 2. Let $0 < \theta, p < \infty$. Let Ω_θ represent the collection of all non-negative measurable functions w on (0, ∞) that are not identically zero and satisfy the condition that for some $t > 0$, the following condition holds:

$$\| w(r) \|_{L_\theta(t,\infty)} < \infty.$$

The space $LM_{p\theta,w}$ is meaningful if and only if $w \in \Omega_\theta$. In particular, the space $LM_{p\theta}^{\lambda}$ is non-trivial if and only if $\lambda > \frac{1}{n}$ when $\theta < \infty$, as well as $\lambda \ge 0$ when $\theta = \infty$.

For a measurable set $\Omega \subset \mathbb{R}^n$ and a non-negative measurable function v defined on Ω, the weighted L_p-space, denoted by $L_{p,v}(\Omega)$ [2], consists of all measurable functions f on Ω that satisfy the condition that

$$\| f \|_{L_{p,v}(\Omega)} = \| vf \|_{L_p(\Omega)} < \infty.$$

It is well established that if $1 \le p \le \infty$, then

$$\| f \|_{LM_{pp,w}} \le \| f \|_{L_{p,W}(\Omega)},$$

and provided that $1 \le p \le \infty$, then

$$\| f \|_{L_{p,W}(\Omega)} \le \| f \|_{LM_{pp,w}},$$

where

$$W(x) = \| w \|_{L_p(|x|,\infty)},$$

for $x \in \mathbb{R}^n$.

The Riesz potential I_α of order α $(0 < \alpha < n)$ plays an important role in harmonic analysis and potential theory, and is defined as follows:

$$I_\alpha f(x) = C_{n,\alpha} \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}}\, dy$$

where $C_{n,\alpha} = \frac{\Gamma\left(\frac{n-\alpha}{2}\right)}{2^\alpha \mathrm{p}^{n/2} \Gamma\left(\frac{\alpha}{2}\right)}$.

It should be noted that the operator I_α is not compact. Therefore, we consider the so-called truncated Riesz potential J_α of order α $(0 < \alpha < n)$, defined as follows:

$$J_\alpha f(x) = C_{n,\alpha} \int_{|y| \le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy,$$

studied in detail in [3]-[10].

Theorem 1. Suppose that $1 \le p \le \infty$, $\alpha > \frac{n}{p}$ and $w \in \Omega_p$. Then the following condition is necessary and sufficient for the boundedness of the truncated Riesz potential J_α from $L_p(\mathbb{R}^n)$ to $LM_{pp,w}$

$$\sup_{r>0} \left(\int_{\mathbb{R}^n \setminus B(0,r)} |x|^{(\alpha-n)p} \|w\|^p_{L_p(|x|,\infty)} dx \right)^{\frac{1}{p}} r^{\frac{n}{p'}} < \infty. \qquad (1)$$

Proof. First we prove the sufficient conditions of Theorem 1

$$J_\alpha f(x) = C_{n,\alpha} \int_{|y| \le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy$$

$$= C_{n,\alpha} \left(\int_{|y| \le \frac{|x|}{2}} \frac{f(y)}{|x-y|^{n-\alpha}} dy + \int_{\frac{|x|}{2} \le |y| \le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right),$$

$$C_{n,\alpha} \left\| \int_{|y| \le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}}$$

$$= C_{n,\alpha} \left\| \int_{|y| \le \frac{|x|}{2}} \frac{f(y)}{|x-y|^{n-\alpha}} dy + \int_{\frac{|x|}{2} \le |y| \le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}}$$

$$\le C_{n,\alpha} \left(\left\| \int_{|y| \le \frac{|x|}{2}} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}} + \left\| \int_{\frac{|x|}{2} \le |y| \le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}} \right),$$

If $|y| \le \frac{|x|}{2}$, then $\frac{1}{|x-y|^{n-\alpha}} \le \frac{b_1}{|x|^{n-\alpha}}$, where b is a positive constant independent of $x \in \mathbb{R}^n$.

$$\left\| \int_{|y| \le \frac{|x|}{2}} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}} = \left\| w(r) \left\| \int_{|y| \le \frac{|x|}{2}} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{L_p(B(0,r))} \right\|_{L_p(0,\infty)}$$

$$\leq c_1 \left\| w(r) \left\| \int_0^{\frac{|x|}{2}} \frac{f(\cdot)}{|x|^{n-\alpha}} d(\cdot) \right\|_{L_p(B(0,r))} \right\|_{L_p(0,\infty)}$$

$$\leq c_2 \sup_{r>0} \left(\int_{\mathbb{R}^n \setminus B(0,r)} |x|^{(\alpha-n)p} \|w\|_{L_p(|x|,\infty)}^p dx \right)^{\frac{1}{p}} r^{\frac{n}{p'}} ||f||_{L_p(\mathbb{R}^n)}.$$

Using Holder's inequality, we get

$$\left\| \int_{\frac{|x|}{2} \leq |y| \leq 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}}^q = \left\| w(r) \left\| \int_{\frac{|x|}{2} \leq |y| \leq 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}} dy \right\|_{L_p(B(0,r))} \right\|_{L_p(0,\infty)}^q$$

$$\leq \left\| w(r) \left\| \left(\int_{\frac{|x|}{2} \leq |y| \leq 2|x|} |f(y)|^p dy \right)^{\frac{1}{p}} \left(\int_{\frac{|x|}{2} \leq |y| \leq 2|x|} \left(\frac{1}{|x-y|^{n-\alpha}} \right)^{p'} dy \right)^{\frac{1}{p'}} \right\|_{L_p(B(0,r))} \right\|_{L_p(0,\infty)}^q$$

$$\leq \left(\int_{\frac{|x|}{2} \leq |y| \leq 2|x|} |f(y)|^p dy \right)^{\frac{q}{p}} \left(\int_{\frac{|x|}{2} \leq |y| \leq 2|x|} \frac{1}{|x-y|^{(n-\alpha)p'}} dy \right)^{\frac{q}{p'}} \|w(r)\|_{L_p(0,\infty)}^{pq}$$

$$\leq c_3 \|f\|_{L_p(\mathbb{R}^n)}^q \left(\frac{|x|^n}{|x-y|^{(n-\alpha)p'}} \right)^{\frac{q}{p'}} \|w(r)\|_{L_p(0,\infty)}^{pq}$$

$$\leq c_4 \|f\|_{L_p(\mathbb{R}^n)}^q \left(\sup_{r>0} \left(\int_{\mathbb{R}^n \setminus B(0,r)} |x|^{(\alpha-n)p} \|w\|_{L_p(|x|,\infty)}^p dx \right)^{\frac{1}{p}} r^{\frac{n}{p'}} \right)^q$$

Now, we will demonstrate that the conditions in Theorem 1 are necessary. Let $k \in \mathbb{Z}$ and $f_k(x) = \chi_{B(0,2^{k-1})}(x)$. Then the following estimates hold:

$$\|J_\alpha f_k\|_{LM_{pp,w}} = \left\| \int_{|y| \leq 2|x|} \frac{f_k(y)}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}} \geq c_5 \left\| \int_{|y| \leq 2^{k-1}} \frac{1}{|x-y|^{n-\alpha}} dy \right\|_{LM_{pp,w}}$$

$$\geq c_6 \left(\|w\|_{L_p(|x|,\infty)}^p \int_{B(0,2^{k+1}) \setminus B(0,2^k)} \frac{|x|^n}{|x-y|^{n-\alpha}} dy \right)^{\frac{1}{p'}}$$

On the other hand, $||f_k||_{L_p(\mathbb{R}^n)} = c_7 \cdot 2^{\frac{nk}{p}}$. Consequently, the boundedness of J_α implies the condition

$$\sup_{k \in \mathbb{Z}} \left(\|w\|_{L_p(|x|,\infty)}^p \int_{B(0,2^{k+1}) \setminus B(0,2^k)} \frac{|x|^n}{|x-y|^{n-\alpha}} dy \right)^{\frac{1}{p'}} < \infty$$

Then $r \in [2^m,\ 2^{m+1})$ for some m, we have

$$\left(\|w\|_{L_p(|x|,\infty)}^p \int_{^cB(0,r)} \frac{1}{|x-y|^{n-\alpha}} dy \right)^{\frac{1}{p'}} r^{\frac{np}{p'}} \leq \left(\|w\|_{L_p(|x|,\infty)}^p \int_{^cB(0,2^m)} \frac{1}{|x-y|^{n-\alpha}} dy \right)^{\frac{1}{p'}} 2^{\frac{(m+1)np}{p'}}$$

$$= c_8 2^{\frac{(m+1)np}{p'}} \sum_{k=m}^{+\infty} \left(\|w\|_{L_p(|x|,\infty)}^p \int_{B(0,2^{k+1}) \setminus B(0,2^k)} \frac{1}{|x-y|^{n-\alpha}} dy \right)^{\frac{1}{p'}}$$

$$\leq c_9 \left(\|w\|_{L_p(|x|,\infty)}^p \int_{B(0,2^{k+1}) \setminus B(0,2^k)} \frac{|x|^n}{|x-y|^{n-\alpha}} dy \right)^{\frac{1}{p'}}$$

Theorem 1 is proven.

Theorem 2. Let $\alpha > \frac{n}{p}$, $1 \le p \le \infty$ functions $w \in \Omega_p$. The truncated Riesz potential J_α acts compactly from spaces $L_p(\mathbb{R}^n)$ to spaces $LM_{pp,w}$ if and only if following conditions

$$\sup_{r>0}\left(\int_{^cB(0,r)} |x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}} r^{\frac{n}{p'}} < \infty$$

and

$$\lim_{a\to 0^+}\sup_{0<r<a}\left(\int_{B(0,a)\setminus B(0,r)} |x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}} r^{\frac{n}{p'}}$$

$$= \lim_{b\to+\infty}\sup_{r\ge b}\left(\int_{B(0,r)} |x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}} (r^n - b^n)^{\frac{1}{p'}} = 0$$

Proof. Let $0 < a < b < \infty$, then we can write

$$J_\alpha f(x) = \int_{|y|\le 2|x|} \frac{f(y)}{|x-y|^{n-\alpha}}dy$$

$$= \chi_{B(0,a)}\int_{|y|\le 2|x|} \frac{\chi_{B(0,2a)}f(y)}{|x-y|^{n-\alpha}}dy + \chi_{B(0,b)\setminus B(0,a)}\int_{|y|\le 2|x|} \frac{\chi_{B(0,4b)}f(y)}{|x-y|^{n-\alpha}}dy$$

$$+\chi_{\,^cB(0,b)}\int_{|y|\le 2|x|} \frac{\chi_{B(0,\frac{b}{2})}f(y)}{|x-y|^{n-\alpha}}dy + \chi_{\,^cB(0,b)}\int_{|y|\le 2|x|} \frac{\chi_{\,^cB(0,\frac{b}{2})}f(y)}{|x-y|^{n-\alpha}}dy$$

Let

$$\overline{k}(x,y) = \chi_{B(0,b)\setminus B(0,a)}(x)\chi_{\{|\cdot|\le 2|x|\}}(y)\frac{1}{|x-y|^{n-\alpha}}$$

Then, taking into account the inequality

$$\int_{|y|\le 2|x|} \frac{1}{|x-y|^{(n-\alpha)p'}}dy \le \frac{b_1|x|^n}{|x|^{(n-\alpha)p'}},$$

where b is a positive constant independent of $x \in \mathbb{R}^n$, we have

$$\left\|\|\overline{k}(x,y)\|_{L_{p'}(\mathbb{R}^n)}\right\|_{LM_{pp,w}(\mathbb{R}^n)}$$

$$\le c_{10}\left(\|w\|^p_{L_p(|x|,\infty)}\int_{B(0,b)\setminus B(0,a)} \frac{|x|^{\frac{p}{p'}}}{|x-y|^{(n-\alpha)p}}dy\right)^{\frac{1}{p}} < \infty$$

From this we conclude that

$$ч_{B(0,b)\setminus B(0,a)}\int_{|y|\le 2|x|} \frac{\chi_{B(0,4b)}f(y)}{|x-y|^{n-\alpha}}dy$$

compact. Similarly we have compactness

$$\chi_{\,^cB(0,b)}\int_{|y|\le 2|x|} \frac{\chi_{B(0,\frac{b}{2})}f(y)}{|x-y|^{n-\alpha}}dy.$$

Taking Theorem 1 into account, we obtain the following estimates:

$$\left\|\chi_{B(0,a)}\int_{|y|\le 2|x|} \frac{\chi_{B(0,2a)}f(y)}{|x-y|^{n-\alpha}}dy\right\| \le b_1\sup_{0<r<a}\left(\int_{B(0,a)\setminus B(0,r)} |x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}} r^{\frac{n}{p'}} < \infty,$$

$$\left\|\chi_{\,^cB(0,b)}\int_{|y|\le 2|x|} \frac{\chi_{\,^cB(0,\frac{b}{2})}f(y)}{|x-y|^{n-\alpha}}dy\right\| \le b_2\sup_{r\ge\frac{b}{2}}\left(\int_{B(0,r)} |x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}}\left(r^n - \left(\frac{b}{2}\right)^n\right)^{\frac{1}{p'}} < \infty$$

Therefore

$$\left\|\int_{|y|\le 2|x|}\frac{f(y)}{|x-y|^{n-\alpha}}dy-\chi_{B(0,b)\setminus B(0,a)}\int_{|y|\le 2|x|}\frac{\chi_{B(0,4b)}f(y)}{|x-y|^{n-\alpha}}dy-\chi_{{}^cB(0,b)}\int_{|y|\le 2|x|}\frac{\chi_{B(0,\frac{b}{2})}f(y)}{|x-y|^{n-\alpha}}dy\right\|$$

$$\le\left\|\chi_{B(0,a)}\int_{|y|\le 2|x|}\frac{\chi\chi_{B(0,2a)}f(y)}{|x-y|^{n-\alpha}}dy\right\|+\left\|\chi_{{}^cB(0,b)}\int_{|y|\le 2|x|}\frac{\chi_{{}^cB(0,\frac{b}{2})}f(y)}{|x-y|^{n-\alpha}}dy\right\|$$

$$\le b_3\sup_{0<r<a}\left(\int_{B(0,a)\setminus B(0,r)}|x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}}r^{\frac{n}{p'}}$$

$$+b_3\sup_{r\ge\frac{b}{2}}\left(\int_{B(0,r)}|x|^{(\alpha-n)p}\|w\|^p_{L_p(|x|,\infty)}dx\right)^{\frac{1}{p}}\left(r^n-\left(\frac{b}{2}\right)^n\right)^{\frac{1}{p'}}<\infty,$$

Consequently, J_α is compact as the limit of compact operators. Theorem 2 is proven.

Corollary. Let $1\le p\le\infty$, $\alpha>\frac{n}{p}$ and $0<\lambda<\infty$. The truncated Riesz potential J_α acts compactly from $L_p(\mathbb{R}^n)$ to LM^λ_{pp} if and only if when the following conditions are met

$$\sup_{0<r<a}\left(\int_{B(0,r)\setminus B(0,a)}|x|^{(\alpha-n)p+1-p\lambda}dx\right)^{\frac{1}{p}}r^{\frac{n}{p'}}<\infty$$

and

$$\lim_{a\to 0^+}\sup_{0<r<a}\left(\int_{B(0,r)\setminus B(0,a)}|x|^{(\alpha-n)p+1-p\lambda}dx\right)^{\frac{1}{p}}r^{\frac{n}{p'}},$$

$$=\lim_{b\to+\infty}\sup_{r\ge b}\left(\int_{B(0,r)}|x|^{(\alpha-n)p+1-p\lambda}dx\right)^{\frac{1}{p}}(r^n-b^n)^{\frac{1}{p'}}=0$$

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О КОМПАКТНОСТИ ПОТЕНЦИАЛА РИССА В ПРОСТРАНСТВАХ ТИПА МОРРИ

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Аннотация. В данной работе исследуется компактность обобщенного усеченного потенциала Рисса в различных вариантах пространств типа Морри. Основное внимание уделено получению необходимых и достаточных условий, обеспечивающих компактность данного оператора. Для этого проведен детальный анализ функциональных свойств обобщенного усеченного потенциала Рисса, что позволило выявить точные условия на параметры пространства, при которых оператор является компактным. В частности, доказаны теоремы, устанавливающие связь между параметрами пространств типа Морри и характеристиками оператора, что позволило расширить существующие результаты, известные ранее лишь в частных случаях. Рассмотренные в работе условия не только являются достаточными, но и оказываются необходимыми, что подчеркивает их точность и оптимальность. Полученные результаты играют важную роль в теории операторов интегрального типа, а также могут быть применены в других областях анализа, таких как теория дифференциальных уравнений и теоретическая физика. Таким образом, проведенное исследование вносит существенный вклад в изучение операторов потенциала Рисса в пространствах типа Морри и открывает новые перспективы для дальнейших исследований в данном направлении. Аналогичный результат для обобщенногоусеченного потенциала Рисса пространствах L_p получены в [10].

Ключевые слова: пространство Морри, пред-компактность, потенциала Рисса, вполне ограниченность, локальные пространства Морри.

МОРРИ ТИПТЕС КЕҢІСТІІГІНДЕГІ РИСС ПОТЕНЦИАЛЫНЫҢ КОМПАКТЫЛЫҒЫ

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Аңдатпа. Бұл жұмыста біз Морри типті кеңістіктердің әртүрлі нұсқаларында жалпыланған қысқартылған Рисс потенциалының жинақылығын зерттейміз. Бұл оператордың жинақылығын қамтамасыз ететін қажетті және жеткілікті жағдайларды алуға басты назар аударылады. Осы мақсатта жалпыланған қысқартылған Рисс потенциалының функционалдық қасиеттеріне егжей-тегжейлі талдау жүргізілді, бұл оператор ықшам болатын кеңістіктің параметрлері бойынша нақты шарттарды анықтауға мүмкіндік берді. Атап айтқанда, Морри типті кеңістіктердің параметрлері мен оператордың сипаттамалары арасында байланыс орнататын теоремалар дәлелденді, бұл бұрын тек ерекше жағдайларда белгілі болған нәтижелерді кеңейтуге мүмкіндік берді. Жұмыста қарастырылған шарттар жеткілікті түрде ғана емес, сонымен қатар олардың нақтылығы мен оңтайлылығына баса назар аударатын қажетті болып табылады. Алынған нәтижелер интегралдық типті операторлар теориясында маңызды рөл атқарады және дифференциалдық теңдеулер теориясы және теориялық физика сияқты талдаудың басқа салаларында да қолданылуы мүмкін. Осылайша, жүргізілген зерттеулер Морри типті кеңістіктердегі Рисс потенциалы операторларды зерттеуге елеулі үлес қосады және осы бағыттағы зерттеулердің жаңа перспективаларын ашады. L_p

кеңістіктерінде жалпыланған қысқартылған Рисс потенциалы алынған нәтижелерге ұқсас нәтижелер [10] жұмысында алынған.

Түйін сөздер: Морри кеңістігі, пре-компакттілік, Рисс потенциалы, толық шектелгендік, локалды Морри кеңістігі.