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METHOD FOR SOLVING LINEAR MATRIX DIFFERENTIAL EQUATIONS SYSTEMS WITH DIFFERENTIATION OPERATOR

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Abstract. This article discusses a linear system of inhomogeneous equations with a differentiation operator for a scalar argument. Cauchy problems for both homogeneous and inhomogeneous systems of equations are studied. The properties of helical characteristics and initial characteristic integrals are established. A new method for studying systems with two differentiation operators is developed based on the transition from a scalar argument to a vector argument. The analytical form of the unique solution to the initial problem for a system with two differentiation operators in a vector-matrix form is found. Integral representations of the unique solution to a system of equations with a differentiation operator in vector form, defined on a cylindrical surface, are obtained for the cases of trivial and periodic initial conditions. As a result of the study, a new method for solving initial value problems for linear homogeneous and inhomogeneous systems with two differentiation operators is developed, based on the transition to a system with one differentiation operator, from which a scheme for studying such systems is provided. The article uses the results and methods of the scientific project "Method of periodic characteristics in the study of oscillations in systems with a diagonal differentiation operator."

Key words: differentiation operator, initial conditions, uniqueness of the solution, periodicity, helical characteristics, linear system, matrix.

Introduction

The article presents the problem of developing a methodology for solving initial problems for systems of linear multiperiod coefficient equations with two-operator double equations. Specific types of such problems, (a) with mutually independent linear parts [2], and (b) with constant coefficients [1], have previously been studied on a plane using a different method. In this study, the general case is considered, and a new method for solving the problem posed on a cylindrical surface is presented. Using this method, a matrix is constructed, and it is substantiated that the initial problems have unique solutions. Additionally, the integral representations of the solutions are provided. The study is based on the method of periodic characteristics presented in [3]. The new method is a further generalization of the method presented in [4]. We note that the reduction of matrices to canonical form [5] and the methods of partial differential equations [6] have contributed to the research.

1. Problem Statement

The study examines a constant matrixant A_0 with a scalar argument (τ, t).

$$D = \frac{\partial}{\partial \tau} + A_0 \frac{\partial}{\partial t}$$

with a differentiation operator $A = [a_{j_k}]_1^n$ matrix operator $f = (f_1, ..., f_n)$ - vector function (θ, ω) - periodic

$$\begin{gathered} Dx = A(\tau,t)x + f(\tau,t), \quad D = \frac{\partial}{\partial \tau} + A_0 \frac{\partial}{\partial t}, \\ A(\tau+\theta, t+\omega) = A(\tau,t) \in C_{\tau,t}^{(0,1)}(R \times R), \\ f(\tau+\theta, t+\omega) = f(\tau,t) \in C_{\tau,t}^{(0,1)}(R \times R), \end{gathered} \tag{1}$$

the given vector function of the system $u(t)=(u_1(t),...,u_n(t))$

$$x\big|_{\tau=\xi}=u(t+\omega)=u(t)\in C_t^{(1)}(R)$$

We consider the problem that satisfies the initial condition. D – It is referred to as a matrix differentiation operator.

We will focus on developing the method for solving this problem in the case where n=2. To do this, we take the constant matrix K and the system (1).

$$x=Ky \tag{2}$$

We transform it with a substitution.. Here, the matrix K should transform the matrix A into its Jordan canonical form J. Therefore, the matrix K is non-singular and

$$K^{-1}A_0K=J \tag{3}$$

will be in the form. Triangular J_1 of Matrix J if the matrix is second-order and the diagonal will be type J_2:

$$J_1=\begin{pmatrix}\nu_0 & 0\\ 1 & \nu_0\end{pmatrix},\quad J_2=\begin{pmatrix}\nu_1 & 0\\ 0 & \nu_2\end{pmatrix}. \tag{4}$$

In general, it can also be $\nu_1=\nu_2$. In this case, the work [1] is provided. We assume that $\nu_1\neq\nu_2$ in this study and that

$$J=\begin{pmatrix}\nu_1 & 0\\ 0 & \nu_2\end{pmatrix},\ \ \nu_1\neq\nu_2 \tag{5}$$

we will consider what happened.
Thus, the operator D is two different

$$\begin{aligned} D_1&=\frac{\partial}{\partial\tau}+\nu_1\frac{\partial}{\partial t},\\ D_2&=\frac{\partial}{\partial\tau}+\nu_2\frac{\partial}{\partial t}\end{aligned} \tag{6}$$

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We assume that here (2) the transformation is given in the form A_0 (5), without entering (1) into the system, and (1) the system $x=(x_1,x_2)$ through the vector

$$Dx=A(\tau,t)x,\quad D=(D_1,D_2) \tag{7}$$

$$A(\tau,t)=[a_{j_k}(\tau,t)]_1^2,\quad A(\tau+\theta,t+\omega)=A(\tau,t)\in C_{\tau,t}^{(0,1)}(R\times R), \tag{8}$$

$$f(\tau,t)=(f_1(\tau,t),f_2(\tau,t)),\quad f(\tau+\theta,t+\omega)=f(\tau,t)\in C_{\tau,t}^{(0,1)}(R\times R) \tag{9}$$

we say that it is given in the form

The initial condition $u(t)=(u_1(t),u_2(t))$ is given through the vector function

$$x\big|_{\tau=\xi}=u(t+\omega)=u(t)\in C_t^{(1)}(R) \tag{7^0}$$

Here, θ,ω let the ratio of the periods be an irrational number. Such numbers are called incommensurable numbers.

We pose the problem of developing a method for solving the initial problem given by conditions (6), (7)–(70), (8), and (9). This problem is not an easy one to solve, as it is examined in detail in [2]. If $A(\tau,t)$ is a constant, then the solution to this problem is presented in the article [3].

2. Helical Characteristics of Differentiation Operators

The differentiation operator is given by the expression

$$D = \frac{\partial}{\partial \tau} + \nu \frac{\partial}{\partial t}, \quad \nu = const > 0, \quad (\tau, t) \in R \times R \tag{10}$$

$R = (-\infty, +\infty)$ - Number axis

Our goal is to determine the descriptor of this operator, given by the expression

$$dt = \nu d\tau \tag{11}$$

which corresponds to a θ-periodic vertical cylindrical surface

To achieve this, when the variable τ changes along the R axis, the τ-variable defined by equation (11) must move with velocity ν compared to the s-variable, passing a straight (temporal) segment of length τ equal to θ. Additionally, t must travel a circular arc of the same length (i.e., equal to θ) along the circumference, and for the phenomenon to be periodic, t must complete one full revolution around the circle.

Therefore, the length of the circle, $2\pi r$, must be equal to $\nu\theta$, that is

$$2\pi r = \nu\theta. \tag{12}$$

Thus, according to equation (12), the radius of the circle is determined by the equality $r = \frac{\nu\theta}{2\pi}$, or the length of the circle must be $C = \nu\theta$

If the circle S is located in the (vw) plane, perpendicular to $u = \tau$ axis, and passes through the origin of the (uvw) Cartesian coordinate system, then the cylinder $Ц$ is aligned along the u-axis.

$$S_\theta : v^2 + (w - r)^2 = r^2 \tag{13}$$

It would be defined by the circle given by the equation. The point (τ, t) along this cylinder (11) moves according to the equation, and its solution passing through the point (ξ, η) is given by the expression x=t (14).

$$t = \nu(\tau - \xi) + \eta \tag{14}$$

If we describe a cylinder using the variables τ and t above, then the parameter τ determines the change in *u* growth, that is, in parallel with the creator of the cylinder, and the parameter *t* indicates the change in the circumference, and therefore the change in the rotation of the cylinder, and it becomes the basis of the parametric equation of the circle S.

So the parametric equations of the cylinder C are in the form

$$Ц : u = \tau, \ v = r \sin\frac{t}{r}, \ w = r - r\cos\frac{t}{r} = r + r\sin\left(\frac{t}{r} - \frac{\pi}{2}\right) \tag{15}$$

If we take into account the expression (14), then the point (τ, t) *(uvw)* in the Cartesian coordinate system, according to the equations (15), we get the trajectory of the propeller movement

$$(\beta), u = \tau - \xi, \ v = r\sin\frac{\nu(\tau - \xi) + \eta}{r}, \ w = r + r\sin\left(\frac{\nu(\tau - \xi)}{r} - \frac{\pi}{2}\right) \tag{16}$$

The coordinates $v = v(\tau)$ and $w = w(\tau)$ in this (16) expression we observe the θ - period ore.

Indeed, considering that according to the expression (12) $\theta = \frac{2\pi r}{\nu}$, then we see that the $v(\tau)$ coordinate satisfies this expression

$$v(\tau + \theta) = r\sin\frac{\nu(\tau + \theta - \xi) + \eta}{r} = r\sin\frac{\nu(\tau + \frac{2\pi r}{\nu} - \xi) + \eta}{r} =$$

$$r\sin\frac{\nu(\tau - \xi) + 2\pi r}{r} = r\sin\left(\frac{\nu(\tau - \xi) + \eta}{r} + 2\pi\right) = r\sin\frac{\nu(\tau - \xi) + \eta}{r} = v(\tau)$$

that is, it is θ-periodic.

In the same way, it can be proved that $w(\tau + \theta) = w(\tau)$.

Thus, the variable *t* (14) is a function of τ depending on the expression, i.e. $t = t(\tau)$. And the Cartesian coordinates of the parameter $t = t(\tau)$ $v(\tau)$ and $w(\tau)$ are θ-periodic functions.

Hence,

$$t(\tau+\theta) = t(\tau), \quad \tau \in R. \tag{17}$$

But the expression (17) cannot be observed from the numerical approximation (14) equality. To overcome such a psychological barrier, we must write that τ is the straight line segment, and *t* is the arc length. To do this, if we take the usual temporal or straight segment τ as it is, then we accept to write the expression (14) in the form

$$t - \eta = dog(\tau - \xi) \tag{18}$$

showing that t is arcuate. The *dog* designation here means the word "arc" (abbreviated, written in Latin).

So, since the circle length S is θ

$$dog(\tau+\theta) = dog\tau \tag{19}$$

We see that the $dog\tau$ function, which converts the straight segment to the price of the circle, is a periodic function and is equal to the length of the periodic circle.

Thus, (18) is equal to (14) written in the language of the operator function, and (14) is equal to (18) written in the term of length. The fact that the function is θ-periodic is given by the equality (19).

There is also an inverse straight line to the $dog\tau = t$ function, that is, a function that converts an arc into a line segment. It has a polynomial function

$$\tau = kest + j\theta = Kest, \quad j \in Z \tag{20}$$

Z is a set of integers.

Kes – a polynomial function that represents the length of a segment or segment, and *kest* is its head value, and it satisfies the condition $0 \le kest < \theta$.

So, write the expression (18) as

$$t = dog(\tau - \xi) + \eta = \beta(\tau, \xi, \eta), \quad \tau \in R \tag{21}$$

and the propeller descriptor of this D operator.

It is an θ - periodic function according to the expression (19).

Solving η from the equation of descriptors (21), we determine the first descriptor integral

$$\eta = \beta(\xi, \tau, t), \ \ (\tau, t) \in R \times S_\theta = Ц \tag{22}$$

It is a flat function that obeys the properties of θ - periodic

$$D\beta(\xi,\tau,t) = 0, \quad \beta(\xi,\xi,t) = t, \tag{23}$$

$$\beta(\xi+\theta,\tau,t) = \beta(\xi,\tau+\theta,t) = \beta(\xi,\tau,t), \tag{24}$$

$$\beta(\xi,\tau,t+\omega) = \beta(\xi,\tau,t) + \omega, \tag{25}$$

$$\beta(\xi,\zeta,\beta(\zeta,\tau,t)) = \beta(\xi,\tau,t) \tag{26}$$

by ξ and τ.

Let's formulate this result obtained in the form of a theorem.

Theorem 1. *The screw descriptors (21) and the first descriptive integrals (22) of the differentiation operator D, the expression of which is in the form (1), are subject to the properties (23)-(26).*

This theorem is fully proven in the work [2], when $\nu = 1$, by a different method. The 1 theorem proved is its generalized form,and it is interpreted in the language of the $dog\tau$ function.

3. Methodology and matrices for studying systems of linear equations with different operators

First, let us dwell on the methods of studying the system of two homogeneous equations

$$\begin{cases} D_1x_1 = a_{11}(\tau,t)x_1 + a_{12}(\tau,t)x_2, D_1 = \dfrac{\partial}{\partial\tau} + \lambda_1\dfrac{\partial}{\partial t} \\ D_2x_2 = a_{21}(\tau,t)x_1 + a_{22}(\tau,t)x_2, D_2 = \dfrac{\partial}{\partial\tau} + \lambda_2\dfrac{\partial}{\partial t} \end{cases} \quad (27)$$

where the coefficients satisfy the conditions

$$a_{j_k}(\tau+\theta, t+\omega) = a_{j_k}(\tau,t) \in C^{(0,1)}_{\tau,t}(R\times S_\theta), \quad (j,k=1,2) \quad (28)$$

and the eigenvalues λ_1 and λ_2 are determined by the condition of diversity $\lambda_1 \neq \lambda_2$.

According to the (1) theorem proved, the descriptors of the operator D_j depend on the values $\nu = \lambda_j, j=1,2$ let it be determined by the equations

$$\frac{dt_j}{d\tau} = \lambda_j, \quad j=1,2; \quad t_j = \beta_j(\tau,\xi,\eta), \quad j=1,2 \quad (29)$$

where the values t_1, t_2 are the names of t in each row.

Thus, t became a vector Value $t=(t_1,t_2)$. According to the theory of descriptors, taking into account that

$$D_jx_j(\tau,t_j)\Big|_{t_j=\beta_j(\tau,\xi,\eta_j)} = \frac{d}{d\tau}x_j(\tau,\beta_j(\tau,\xi,\eta_j), \quad j=1,2 \quad (30)$$

each line has its own descriptor, taking σ instead of τ, we write the system

$$\frac{d}{d\sigma}x_j(\sigma,\beta(\sigma,\xi,\eta_j)) = \sum\nolimits_{k=1}^{2} a_{j_k}(\sigma,\beta_j(\sigma,\xi,\eta_j)x_j(\sigma,\beta_j(\sigma,\xi,\eta_j))),$$
$$\xi \xrightarrow{\sigma} \tau \quad (31)$$

and note that the variable σ here varies from ξ to τ. (ξ,η_j) is the initial set of descriptors, i.e. the preliminary position of the (τ,t_j) vector.

So, from the differential system (31)

$$x_j(\sigma,\beta_j(\sigma,\xi,\eta_j)) = u_j(\eta_j) + \int_\xi^\tau \sum_{k=1}^{2} a_{j_k}(\sigma,\beta_j(\sigma,\xi,\eta_j))x_k(\sigma,\beta_j(\sigma,\xi,\eta_j))d\sigma, \quad j=1,2 \quad (32)$$

we got an integral system.

Now let's move from this scalar writing to the vector-matrix writing form.

To do this, we need to move from all coordinate quantities to vector quantities, from Element quantities to matrices and determine the order of execution of operators.

In accordance with this, without breaking the already formed familiar designations, two - coordinate vectors $(\tau,t),(\xi,\eta_1),(\xi,\eta_2), \ x=(x_1,x_2), \ u=(u_1,u_2), \ \eta=(\eta_1,\eta_2), \ t=(t_1,t_2), \ \beta=(\beta_1,\beta_2)$ and $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ were obtained, given by The matrix and the ξ,σ,τ scalars, the vector operator $D=(D_1,D_2)$. Let's solve the problem of writing expressions (27)-(32), given by arithmetic differential, integral and composite operations, in such a way that other mathematicians understand and understand it, so that they can easily describe the often used method of gradual approximation.

The main feature of this situation, the previously unestablished rule, was the coordination issue of ensuring that each line's characteristic descriptor remains in its place. This issue arises when multiplying a matrix by another matrix or a matrix by a vector, due to the involvement of all rows in the matrix interacting with each row. When integrals are repeated, a new composition is introduced first.

The raised issue had previously been solved using projection operators. Бірақ, ол әлі толық әкетілмеген. Unless in specific cases (e.g., with constant coefficients), we notice that applying it in general is difficult.

The way to solve this issue now seems to have been found. It is the resolution of the composite operation after all arithmetic operations, followed by the execution of differentiation or integration. The algorithm of operations: 1) perform arithmetic operations, 2) apply composite operations, then 3) adopt the rule for using integral or differential operations, and 4) both integration and differentiation should be carried out in terms of descriptors, with the results being expressed in terms of the initial descriptor integrals. This can be referred to as the hierarchy of operations.

A composite operation is the process of substituting the argument of a function with another argument, meaning it is the operation of introducing complex functions.

For example, the substitution of the (τ,t) arguments of the function x into the vector-function $x(\tau,t)$, i.e., the transformation from $x(\tau,t)$ { $x(\sigma,\beta(\sigma,\tau,t))$, is written in the form

$$x((\tau,t)\,\mathrm{o}(\sigma,\beta(\sigma,\tau,t)))=x(\sigma,\beta(\sigma,\tau,t)) \qquad (33_1)$$

Here, the small circle o is the symbol indicating which quantity is substituted by which other quantity.

Now, let's adopt the following notation suitable for our case:

$$Dx=\begin{pmatrix}D_1\\D_2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}D_1x_1\\D_2x_2\end{pmatrix}, \qquad (33_2)$$

$$x(\tau,t)=\begin{pmatrix}x_1\\x_2\end{pmatrix}\begin{pmatrix}(\tau,t_1)\\(\tau,t_2)\end{pmatrix}=\begin{pmatrix}x_1(\tau,t_1)\\x_2(\tau,t_2)\end{pmatrix}, \qquad (33_3)$$

$$A(\tau,t)=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}(\tau,t)=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\begin{pmatrix}(\tau,t_1)\\(\tau,t_2)\end{pmatrix}=\begin{pmatrix}a_{11}(\tau,t_1)&a_{12}(\tau,t_1)\\a_{21}(\tau,t_2)&a_{22}(\tau,t_2)\end{pmatrix}, \qquad (33_4)$$

$$A(\tau,t)x(\tau,t)=(Ax)(\tau,t)=\begin{pmatrix}a_{11}x_1+a_{12}x_2\\a_{21}x_1+a_{22}x_2\end{pmatrix}\begin{pmatrix}(\tau,t_1)\\(\tau,t_2)\end{pmatrix}=\begin{pmatrix}a_{11}(\tau,t_1)x_1(\tau,t_1)+a_{12}(\tau,t_1)x_2(\tau,t_1)\\a_{21}(\tau,t_2)x_1(\tau,t_2)+a_{22}(\tau,t_2)x_2(\tau,t_2)\end{pmatrix}=$$
$$(Ax)(\tau,t). \qquad (33)$$

Thus, we need to construct the mappings of vector and matrix functions and their arguments.

Thus, the system (27) is written in vector-matrix form with the acceptations (33_1)-(33_5) as

$$Dx=(Ax)(\tau,t) \qquad (34)$$

Meanwhile, the expression (28) retains the previous form of

$$A(\tau+\theta,t+\omega)=A(\tau,t) \qquad (35)$$

Similarly, the expressions (29) also retain their previous forms of

$$\frac{dt}{d\tau}=\lambda,\quad t=\beta(\tau,\xi,\eta) \qquad (36)$$

The relationship of the differential operators (30) is described by

$$Dx(\tau,t)\big|_{t=\beta(\tau,\xi,\eta)}=\frac{d}{d\tau}x(\tau,\beta(\tau,\xi,\eta)) \qquad (37)$$

which also remains unchanged.

The simple differential system (31) is presented in the form

$$\frac{d}{d\sigma}x(\sigma,\beta(\sigma,\xi,\eta))=(Ax)(\sigma,\beta(\sigma,\xi,\eta)) \qquad (38)$$

Also, if we take the function

$$u(\eta)=\begin{pmatrix}u_1(\eta_1)\\u_2(\eta_2)\end{pmatrix}$$

the integral system (32) is presented as

$$x(\sigma,\beta(\sigma,\xi,\eta)) = u(\eta) + \int_{\xi}^{\tau}(Ax)(\sigma,\beta(\sigma,\xi,\eta))d\sigma \tag{39}$$

As a result of the integration, in order to show what function is described by the expression (39), we consider that

$$\beta(\sigma,\xi,\beta(\xi,\tau,t)) = \beta(\sigma,\tau,t)$$

is the initial integral and that the complex function becomes

$$\eta = \beta(\xi,\tau,t)$$

Therefore, we transition to the expression

$$x(\tau,t) = u(\beta(\xi,\tau,t)) + \int_{\xi}^{\tau}(Ax)(\sigma,\beta,(\sigma,\tau,t))d\sigma \tag{40}$$

If we replace $x(\tau,t)$ in the expression (40) with the $X(\tau,t)$ matrix, then considering

$$X\big|_{\tau=\xi} = V(\eta)\,,$$

it can be written as

$$X(\tau,t) = V(\beta(\xi,\tau,t)) + \int_{0}^{\tau}(AX)(\sigma,\beta(\sigma,\tau,t))d\sigma \tag{41}$$

In the special case where *V*=*E* is the identity matrix, we obtain the expression

$$X(\tau,t) = E + \int_{0}^{\tau}(AX)(\sigma,\beta(\sigma,\tau,t))d\sigma \tag{42}$$

from the equation (41).

Approximations should be expressed in terms of (τ,t).

Now, let's provide the methodology for constructing the matrix using the iterative approximation method.

To do this, we open the unit matrix E as the zero approximation $X^{(0)}$, that is, from the approximations

$$X^{(0)} = E,\, X^{(k)}_{(\tau,t)} = E + \int_{\xi}^{\tau}(AX^{(k-1)})(\sigma,\beta(\sigma,\tau,t))d\sigma, \quad (k = 1,2,...) \tag{43}$$

and get the first approximation

$$X^{(1)}_{(\tau,t)} = E + \int_{\xi}^{\tau}A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0$$

$$X^{(2)}_{(\tau,t)} = E + \int_{\xi}^{\tau}A(\sigma_1,\beta(\sigma_1,\tau,t))\cdot X^{(1)}_{(\tau,t)}(\sigma_0,\beta(\sigma_0,\sigma_1,\beta(\sigma_1,\tau,t))) =$$

$$= E + \int_{\xi}^{\tau}A(\sigma_2,\beta(\sigma_2,\tau,t))\int_{\xi}^{\sigma_2}[E + A(\sigma_0,\beta(\sigma_0,\sigma_1,\beta(\sigma_1,\tau,t)))]d\sigma_1 =$$

$$= \beta(\sigma_0,\sigma_1,\beta(\sigma_1,\tau,t)) = \beta(\sigma_0,\tau,t) =$$

$$= E + \int_{\xi}^{\tau}A(\sigma_1,\beta(\sigma_1,\tau,t))d\sigma_1 + \int_{\xi}^{\tau}A(\sigma_1,\beta(\sigma_1,\tau,t))\int_{\xi}^{\sigma_1}A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0 =$$

$$= E + \int_{\xi}^{\tau}A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0 + \int_{\xi}^{\tau}d\sigma_1\int_{\xi}^{\sigma_1}A(\sigma_1,\beta(\sigma_1,\tau,t))A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0$$

$$\dots$$

$$X^{(k)}_{(\tau,t)} = E + \int_{\xi}^{\tau}A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0 + \int_{\xi}^{\tau}d\sigma_1\int_{\xi}^{\sigma_1}A(\sigma_1,\beta(\sigma_1,\tau,t))A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0 +$$
$$+ ... + \int_{\xi}^{\tau}d\sigma_{k-1}\int_{\xi}^{\sigma_{k-1}}d\sigma_{k-2}...\int_{\xi}^{\sigma_k}A(\sigma_{k-1},\beta(\sigma_{k-1},\tau,t))...A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0 \tag{44}$$

$$\dots$$

We obtain the expressions.

Now we estimate the approximations (44) and determine the inequalities

$$\left|X^{(0)}\right| = |E| = 1, \left|X^{(1)}_{(\tau,t)}\right| \le 1 + \|A\||\tau-\xi|,$$

$$\left|X^{(2)}_{(\tau,t)}\right| \le 1 + \|A\||\tau-\xi| + \left|\int_{\xi}^{\tau}\|A\|^2|\sigma_1-\xi|d\sigma_1\right| \le 1 + \|A\||\tau-\xi| + \frac{\|A\|^2|\tau-\xi|^2}{2!},..., \left|X^{(k)}_{(\tau,t)}\right| \le$$

$$1+\frac{\|A\||\tau-\xi|}{1!}+\frac{\|A\|^2|\tau-\xi|^2}{2!}+...+\frac{1}{k!}\|A\|^k|\tau-\xi|^k, k=0,1,2,... \tag{45}$$

Therefore, from the expression (45) we see that

$$\left|X^{(k)}_{(\tau,t)}\right|\le e^{\|A\||\tau-\xi|} \tag{46}$$

Now, let's evaluate the differences between the two related approximations. Then it is possible to make sure that

$$\left|X^{(1)}_{(\tau,t)}-X^{(0)}\right|\le\left|\int_\xi^\tau A(\sigma_0,\beta(\sigma_0,\tau,t))d\sigma_0\right|\le\|A\||\tau-\xi|,$$

$$\left|X^{(2)}_{(\tau,t)}-X^{(1)}_{(\tau,t)}\right|\le\frac{\|A\|^2|\tau-\xi|^2}{2!},$$

$$\dots$$

$$\left|X^{(k)}_{(\tau,t)}-X^{(k-1)}_{(\tau,t)}\right|\le\frac{\|A\|^2|\tau-\xi|^k}{k!} \tag{47}$$

$$\dots$$

by the method of full mathematical induction.

Therefore, we know that for any number $\Delta>0$, from the expression

$$X^{(k)}_{(\tau,t)}=X^{(0)}+\sum_{j=1}^{k}[X^{(j)}_{(\tau,t)}-X^{(j-1)}_{(\tau,t)}]$$

when $|\tau-\xi|\le\Delta$, it is expressed by the series

$$\lim_{k\to\infty}X^{(k)}_{(\tau,t)}=E+\sum_{j=1}^{\infty}[X^{(j)}_{(\tau,t)}-X^{(j-1)}_{(\tau,t)}] \tag{48}$$

when $k\to\infty$.

On the basis of this (48) row (47) estimation, we get that

$$|E|+\sum_{j=1}^{\infty}\left|X^{(j)}_{(\tau,t)}-X^{(j-1)}_{(\tau,t)}\right|\le1+\sum_{j=1}^{\infty}\frac{\|A\|^j|\tau-\xi|^j}{j!}\le1+\sum_{j=1}^{\infty}\frac{\|A\|^j\Delta^j}{j!}=e^{\|A\|\Delta}$$

satisfies.

So, if the (48) series is $|\tau-\xi|\le\Delta$, the absolute is set in a uniform.

If we denote the sum of the series as $X(\tau,t)$, then from the extreme (48) we get the equality

$$\lim_{k\to\infty}X^{(k)}_{(\tau,t)}=X(\tau,t) \tag{49}$$

From these(42)-(49) expressions, i.e., in the method of constructing a matrices(33_1) - (33_5), we found that the receptions function perfectly, as in a single-operator system. Finally, to obtain the matrices of the system (27) - (28), we note that from the expression (29) we write the scalar value $t\in R$ as a vector

$$\overset{o}{t}=\begin{pmatrix}t\\t\end{pmatrix}=\begin{pmatrix}t_1\\t_2\end{pmatrix}\subset R^2 \tag{50}$$

corresponding to each line of the system. Where the sign = is composite equality. Therefore, we note that in the expression (49) *t* is in the form (50), that is, according to it,

$$X(\tau,t)=[x_{j_k}(\tau,t)]_1^2=\begin{bmatrix}x_{11}(\tau,t_1) & x_{12}(\tau,t_1)\\x_{21}(\tau,t_2) & x_{22}(\tau,t_2)\end{bmatrix} \tag{51}$$

is represented in the form (51). Therefore, we note that in the expression (49) T is in the form (50), that is, according to it,

$$X(\tau,t)=[x_{j_k}(\tau,t)]_1^2=\begin{bmatrix}x_{11}(\tau,t_1) & x_{12}(\tau,t_1)\\x_{21}(\tau,t_2) & x_{22}(\tau,t_2)\end{bmatrix} \tag{51}$$

is represented in the form Then, if the (τ,t)-scalar argument of the system (27) is given in vector-matrix form as

$$Dx^{\circ} = A^{\circ}(\tau,t)x^{\circ}, \quad D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}, \quad A^{\circ}(\tau,t) = \begin{bmatrix} a_{11}(\tau,t) & a_{12}(\tau,t) \\ a_{21}(\tau,t) & a_{22}(\tau,t) \end{bmatrix}, \quad (\tau,t) \in R \times R \qquad (52)$$

notation, and considering that $t_1=t_2=t$ in the expression (51), we will familiarize ourselves with the scalar argument, i.e., the $(\tau,t) \in R\times R$ argument, from the expression

$$X(\tau,t) = X\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix}$$

in the form of a matrix

$$X^{\circ}(\tau,t) = X\begin{pmatrix} (\tau,t) \\ (\tau,t) \end{pmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\begin{pmatrix} (\tau,t) \\ (\tau,t) \end{pmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}(\tau,t), (\tau,t) \in R \times R \qquad (53)$$

Thus, in this section of the study, we transitioned from the (τ,t) scalar argument system to the $\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix} = ((\tau,t_1),(\tau,t_2))$ vector-argument system and became acquainted with the research method based on assumptions (33_1)-(33_5), successfully applying the new method for constructing the matrix of the linear system.

Theorem-Statement 2. *By transitioning a system with two differentiation operators from a scalar argument to a vector argument, a new research method was developed based on the arithmetic, compositional, and analytical operations hierarchy (33_1)-(33_5), and the developed method was introduced through the construction of the matrix of the linear system (27) and the vector-matrix equation (52) (53).*

Theorem 1) states the new method, formulating and summarizing it, and then with this method, 2) it determines the form of the matrix. Therefore, it is referred to as a conceptual theorem. The use of limits, differentiation, and integration operators is explained by the term analytical operations.

4. Solving two-operator linear systems on a cylindrical surface.

Consider the problem of finding a solution to the equation

$$\begin{gathered} Dx = A(\tau,\mathbf{t})x, \quad (\tau,\mathbf{t}) \in R \times S_\theta, \quad D = (D_1, D_2), \\ A(\tau,t) = [a_{j_k}(\tau,\mathbf{t})]_1^2, \quad A(\tau+\theta, \mathbf{t}+\omega) = A(\tau,\mathbf{t}) \in C^{(0,1)}_{\tau,\mathbf{t}}(R \times S_\theta) \end{gathered} \qquad (54)$$

of the vector-matrix type

$$x\big|_{\tau=\xi} = u(\mathbf{t}) = u(\mathbf{t}+\omega) \in C^{(1)}_{\mathbf{t}}(S_\theta) \qquad (54^0)$$

of a linear two-operator (τ,t) scalar argument (27) - (28) system (52). Here $x=(x_1,x_2)$ an unknown vector is a function whose variable t indicates that it is a scalar quantity

To do this, we use the method according to theorem 2-statement. For this purpose,instead of the $(\tau,\mathbf{t})$ scalar arguments, we introduce the vector $t=(t_1,t_2)$, defined by the Equality $t_1 = t_2 = \mathbf{t}$, and consider the vector argument

$$((\tau,t_1),(\tau,t_2)) = (\tau,t) \qquad (55)$$

(i.e. (τ,t_1) - the arguments of the first line, (τ,t_2) - the arguments of the second line) as the following system

$$Dx = (Ax)(\tau,t), \quad D = (D_1, D_2), \quad A\begin{pmatrix} (\tau+\theta, t_1+\omega) \\ (\tau+\theta, t_2+\omega) \end{pmatrix} = A\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix} = A(\tau,t) \in C^{(0,1)}_{\tau,t}(R \times S_\theta) \qquad (56)$$

The initial condition is as

$$x\big|_{\tau=\xi} = u(t) = u(t+\omega) \in C^{(1)}_t(S_\theta), \quad t = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \qquad (56^0)$$

Now we will understand the (τ,t) argument in the sense of (55) until we turn it back to the problem (54)-(54^0) and, in accordance with the perception (33_1)-(33_5), we will conduct a study using arithmetic, compositional and analytical operations, in this hierarchical order. This method draws on the scheme of the single-operator system approach used earlier, we only need to understand the use of operations in the sense of (33_1) - (33_5).

In the general case, the equations with the D operator should be simply set to the $\frac{d}{d\tau}$ operator state, and then the same characteristic-descriptive equations should be normalized in the form of a single system, grouped by $(\sigma,\beta_j(\sigma,\xi,\eta)),(\xi,\eta)\in R\times S_\theta = Ц,\quad \xi \xrightarrow{\sigma} \tau$, that is, grouped according to the identity of the characteristics of the given system.

In our case, the descriptors of the two equations can be 1) the same or 2) different. In the same 1) case, the system is single-operator, and its integration methods are known, given in the work [3], developed in the work [4].

If in the case of 2), then each equation of the system is determined along its own descriptor, and it must be integrated according to the new methodology in this study.

A thin hyperbolic situation, that is, a system with different descriptions, is integrated from the described method. According to this approach, the system (56) is written in the form

$$\frac{d}{d\sigma}x(\sigma,\beta(\sigma,\xi,\eta)) = (Ax)(\sigma,\beta(\sigma,\xi,\eta)),\qquad A(\sigma+\theta,\beta+\omega) = A(\sigma,\beta)\in C^{(0,1)}_{\sigma,\beta}(R\times S_\theta) \tag{57}$$

$$x\big|_{\sigma=\xi} = u(\eta+\omega) = u(\eta)\in C^{(1)}_\eta(S_\theta),\xi \xrightarrow{\sigma} \tau,(\xi,\eta)\in Ц \tag{57^0}$$

and according to the expressions designations (331)-(335), we must understand in the form of dependencies

$$\eta = \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix},\quad \beta = \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix},\quad x = \begin{pmatrix}x_1\\ x_2\end{pmatrix},\quad u = \begin{pmatrix}u_1\\ u_2\end{pmatrix},\quad A = \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix},$$

$$(Ax) = \begin{pmatrix}a_{11}x_1 + a_{12}x_2\\ a_{21}x_1 + a_{22}x_2\end{pmatrix} = \begin{pmatrix}f_1\\ f_2\end{pmatrix},\quad (\sigma,\beta) = \begin{pmatrix}(\sigma,\beta_1)\\ (\sigma,\beta_2)\end{pmatrix},\quad (\xi,\eta) = \begin{pmatrix}(\xi,\eta_1)\\ (\xi,\eta_2)\end{pmatrix},$$

$$(Ax)(\sigma,\beta(\sigma,\xi,\eta)) = \begin{pmatrix}a_{11}x_1 + a_{12}x_2\\ a_{21}x_1 + a_{22}x_2\end{pmatrix}\begin{pmatrix}(\sigma,\beta_1(\sigma,\xi,\eta_1))\\ (\sigma,\beta_2(\sigma,\xi,\eta_2))\end{pmatrix} =$$

$$= \begin{pmatrix}(a_{11}x_1 + a_{12}x_2)(\sigma,\beta_1(\sigma,\xi,\eta_1))\\ (a_{21}x_1 + a_{22}x_2)(\sigma,\beta_2(\sigma,\xi,\eta_2))\end{pmatrix} = \begin{pmatrix}f_1(\sigma,\beta_1(\sigma,\xi,\eta_1))\\ f_2(\sigma,\beta_2(\sigma,\xi,\eta_2))\end{pmatrix}$$

and $f_j = \sum_{k=1}^{2} a_{j_k}x_k,\ j = 1,2$. The vector argument of the matrix is written in parallel, but it should be noted that it is not multiplied.

According to theorem-statement 2, the solution of the initial problem (57)-(570) is given by the expression

$$x(\sigma,\beta(\sigma,\xi,\eta)) = X(\sigma,\beta(\sigma,\xi,\eta))u(\eta) \tag{58}$$

in the term descriptors. From this we proceed to the integral of the first descriptors by $\eta = \beta(\xi,\tau,t)$, taking into account the expression

$$x(\sigma,\beta(\sigma,\xi,\beta(\xi,\tau,t))) = X(\sigma,\beta(\sigma,\xi,\beta(\xi,\tau,t)))u(\beta(\xi,\tau,t))$$

from it the image

$$x(\sigma,\beta(\sigma,\tau,t)) = X(\sigma,\beta(\sigma,\tau,t))u(\beta(\xi,\tau,t))$$

by the group property, and therefore, if $\sigma = \tau$, then $\beta(\tau,\tau,t) = t$, then

$$x(\tau,t) = X(\tau,t)u(\beta(\xi,\tau,t)) \tag{59}$$

(56)-(56^0) we get a solution to the problem. The solution is vector-argument, i.e. $t=\begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$.

Now we move on to the case $t=\begin{pmatrix} \mathbf{t} \\ \mathbf{t} \end{pmatrix}=\mathbf{t}$, i.e. $t_1=t_2=\mathbf{t}$, and from the expression (59) we get the solution of the problem (54)-(54^0) in the form

$$x(\tau,\mathbf{t})=X(\tau,\mathbf{t})u(\beta(\xi,\tau,t)) \qquad (60)$$

Theorem 3. *The only solution to the initial problem (54) - (54^0) is determined in the form (60) obtained by the expression (59).*

We also remind you that the full proof of the theorem is based on the above using the expressions (54)-(60). The loneliness of the solution is easily determined by the reverse approach.

Using the same method, let's determine the solution of the initial condition problem

$$x\big|_{\tau=\xi}=u(\mathbf{t})=u(\mathbf{t}+\omega)\in C_{\mathbf{t}}^{(1)}(S_\theta) \qquad (61^0)$$

for the system

$$\begin{gathered} Dx=A(\tau,\mathbf{t})x+f(\tau,\mathbf{t}), \quad D=(D_1,D_2), \\ A(\tau+\theta,\mathbf{t}+\omega)=A(\tau,\mathbf{t})\in C_{\tau,\mathbf{t}}^{(0,1)}(R\times S_\theta), \\ f(\tau+\theta,\mathbf{t}+\omega)=f(\tau,\mathbf{t})\in C_{\tau,\mathbf{t}}^{(0,1)}(R\times S_\theta) \end{gathered} \qquad (61)$$

with the unknowns $x=(x_1,x_2)$ in the linear non-homogeneous case.

In the new method, we take congruent variables $t_1=t_2=\mathbf{t}$ and write the problem (61)-(61^0) as

$$\begin{gathered} Dx=(Ax)(\tau,t)+f(\tau,t), \quad (\tau,t)=\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix}, \\ A\begin{pmatrix} (\tau+\theta,t_1+\omega) \\ (\tau+\theta,t_2+\omega) \end{pmatrix}=A\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix}=A(\tau,t)\in C_{\tau,t}^{(0,1)}(R\times S_\theta), \\ f\begin{pmatrix} (\tau+\theta,t_1+\omega) \\ (\tau+\theta,t_2+\omega) \end{pmatrix}=f\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix}=f(\tau,t)\in C_{\tau,t}^{(0,1)}(R\times S_\theta), \end{gathered} \qquad (62)$$

$$x\big|_{\tau=\xi}=u\begin{pmatrix} t_1+\omega \\ t_2+\omega \end{pmatrix}=u\begin{pmatrix} t_1 \\ t_2 \end{pmatrix}=u(t)\in C_t^{(0,1)}(S_\theta) \qquad (62^0)$$

In general, we draw attention to the fact that the expression

$$f(\tau,t)=\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix}=\begin{pmatrix} f_1(\tau,t_1) \\ f_2(\tau,t_2) \end{pmatrix}$$

is called the cartesian product of the vector-function *f* and the vector-argument (τ,t). Therefore, the cartesian product is equivalent to registering the corresponding coordinates of two vectors and writing them together.

Further, from the problem (62)-(62^0), we move along the descriptions to the differential equations

$$\begin{pmatrix} (\tau,t_1) \\ (\tau,t_2) \end{pmatrix}=(\tau,t)=(\sigma,\beta(\sigma,\xi,\eta))=\begin{pmatrix} \sigma,\beta_1(\sigma,\xi,\eta_1) \\ \sigma,\beta_2(\sigma,\xi,\eta_2) \end{pmatrix}$$

and determine the solution of the initial problem

$$\begin{gathered} \frac{dx(\sigma,\beta(\sigma,\xi,\eta))}{d\sigma}=(Ax)(\sigma,\beta(\sigma,\xi,\eta)) \\ A(\sigma+\theta,\beta+\omega)=A(\sigma,\beta)\in C_{(\tau,\beta)}^{(0,1)}(R\times S_\theta), \\ f(\sigma+\theta,\beta+\omega)=f(\sigma,\beta)\in C_{(\tau,\beta)}^{(0,1)}(R\times S_\theta), \end{gathered} \qquad (63)$$

$$x\big|_{\sigma=\xi}=u(\eta+\omega)=u(\eta)\in C_\eta^{(1)}(S_\theta) \qquad (63^0)$$

To do this, let's first look for a solution that satisfies the initial condition in the simple case of universal revelation

$$u\big|_{\sigma=\xi} = 0 \qquad (63^*)$$

It is known from the theory of differential equations that such a solution is given by the expression

$$x^{o}(\sigma,\beta(\sigma,\xi,\eta)) = X(\sigma,\beta(\sigma,\xi,\eta))\int_{\xi}^{\sigma}(Xf^{-1})(s,\beta(s,\xi,\eta))ds \qquad (64)$$

for the problem(64)-(64*) [6].

Thus, a characteristic feature of linear systems is that the solution of the initial problem (63)-(63^0) consists of the sum of the solution of homogeneous systems (57)-(57^0) and the solution of (63)-(63*). Therefore, along the descriptors are represented by the expression

$$x(\sigma,\beta(\sigma,\xi,\eta)) = X(\sigma,\beta(\sigma,\xi,\eta))u(\eta) + X(\sigma,\beta(\sigma,\xi,\eta))\int_{\xi}^{\sigma}(X^{-1}f)(s,\beta(s,\xi,\eta))ds \quad (65)$$

If we take into account that $\eta = \beta(\xi,\tau,t)$, then the solutions (64) and (65), based on the equality $\beta(\sigma,\xi,\beta(\xi,\tau,t)) = \beta(\sigma,\tau,t)$, form

$$x^*(\sigma,\beta(\sigma,\tau,t)) = X(\sigma,\beta(\sigma,\tau,t))\int_{\xi}^{\sigma}(X^{-1}f)(s,\beta(s,\tau,t))ds, \qquad (66)$$

$$x(\sigma,\beta(\sigma,\tau,t)) = X(\sigma,\beta(\sigma,\tau,t))u(\beta(\xi,\tau,t)) + X(\sigma,\beta(\sigma,\tau,t))\int_{\xi}^{\sigma}(X^{-1}f)(s,\beta(s,\tau,t))ds \quad (67)$$

along the first descriptive integrals.

Further, if $\sigma = \tau$, then we take into account that $\beta(\tau,\tau,t) = t$, and from the expressions (66) and (67) we get the corresponding introductions

$$x^*(\tau,t) = X(\tau,t)\int_{\xi}^{\tau}(X^{-1}f)(s,\beta(s,\tau,t))ds \qquad (68)$$

$$x(\tau,t) = X(\tau,t)u(\beta(\xi,\tau,t)) + X(\tau,t)\int_{\xi}^{\tau}(X^{-1}f)(s,\beta(s,\tau,t))ds \qquad (69)$$

These (68) and (69) solutions are the solutions to the initial problems (62)-(62^0)-(63*) and (62)-(62^0), respectively.

From these solutions, we can see that if we project the variable $t = \begin{pmatrix}\mathbf{t}\\ \mathbf{t}\end{pmatrix}$, i.e. t, to the vector state $P\mathbf{t} = \begin{pmatrix}t_1\\ t_2\end{pmatrix} = t$, in the case of $t_1 = t_2 = \mathbf{t}$, then the expressions

$$x^*(\tau,\mathbf{t}) = X(\tau,\mathbf{t})\int_{\xi}^{\tau}(X^{-1}f)(s,\beta(s,\tau,\mathbf{t}))ds, \qquad (70^0)$$

$$x(\tau,\mathbf{t}) = X(\tau,\mathbf{t})u(\beta(\xi,\tau,\mathbf{t})) + X(\tau,\mathbf{t})\int_{\xi}^{\tau}(X^{-1}f)(s,\beta(s,\tau,\mathbf{t}))ds \qquad (70)$$

give a solution to the initial problem (61)-(61^0) without using the projection operation, given that $\mathbf{t} = P^{-1}t$ reduces the vector state to a scalar state.

Thus, the following theorems have been proven.

Theorem 4. *The unique solution of the given initial problem (61)-(610)-(63*) on the cylindrical surface is expressed by the formula (70^0).*

The uniqueness of the solution was proven using the method of contradiction. The following Theorem 5, which unites Theorems 3 and 4, is stated as follows.

Theorem 5. The unique solution of the given initial problem (61)-(61^0) on the cylindrical surface is expressed in the form (70).

This theorem is, on one hand, a general theorem, and on the other hand, a consequence of Theorems 3 and 4.

The significant achievement of the research is the development of a new method that transforms the solutions of the initial problems of two-operator linear systems into the form of the solution scheme for single-operator systems.

Conclusion.

In this work, the uniqueness of the solution was proven using the method of contradiction. A significant achievement of the research is that the solutions of the initial problems of two-operator linear systems were transformed into a form of the solution scheme for single-operator systems, and attention was paid to the development of a new method.

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СЫЗЫҚТЫ МАТРИЦАЛЫҚ ДИФФЕРЕНЦИАЛДАУ ОПЕРАТОРЛЫ ТЕҢДЕУЛЕР ЖҮЙЕЛЕРІН ШЕШУДІҢ ӘДІСІ

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Андатпа. Мақалада скалярлық аргумент бойынша дифференциалдау операторы бар біртекті емес теңдеулердің сызықтық жүйесі қарастырылады. Біртекті және біртекті емес теңдеулер жүйелері үшін Коши мәселелері зерттелген. Осыған сәйкес, бұрандалық сипаттамалар мен бастапқы сипаттамалық интегралдардың қасиеттері анықталған. Скалярлық аргументтен векторлық аргументке өту негізінде екі дифференциалдау операторы бар жүйелерді зерттеудің жаңа әдісі әзірленген. Вектор-матрица түріндегі берілген екі дифференциалдау операторы бар жүйенің бастапқы мәселесі үшін бірегей шешімнің аналитикалық түрі табылған. Цилиндрлік бетте берілген дифференциалдау операторы бар теңдеулер жүйесінің бірегей шешімінің интегралдық көріністері алынды, мұнда бастапқы жағдайдың тривиалды және периодтық жағдайлары қарастырылған. Мақалада дифференциалдық теңдеулер теориясындағы дифференцирлеу операторлары бар зерттеулердің мүмкін бағыттары да талқыланған. Зерттеу нәтижесінде екі дифференциалдау операторы бар сызықтық біртекті және біртекті емес жүйелер үшін бастапқы мәселелерді зерттеудің жаңа әдісі әзірленді, ол да бір операторы бар дифференциалдау жүйесіне өту негізінде жасалған және осындай жүйелерді зерттеу схемасы ұсынылған. Мақалада тағы «Диагональ бойынша дифференциалдау операторы бар жүйелердегі тербелістерді зерттеудегі периодтық сипаттамалар әдісі» ғылыми жобасында нәтижелері мен әдістері алынып пайдаланылған.

Түйін сөздер: дифференциалдау операторы, бастапқы шарт, шешімнің жалғыздығы, периодтар, винттік сипаттауыштар, сызықты жүйелер, матрица.

МЕТОД РЕШЕНИЯ СИСТЕМ ЛИНЕЙНЫХ МАТРИЧНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ОПЕРАТОРОМ ДИФФЕРЕНЦИРОВАНИЯ

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Аннотация. В данной статье рассматривается линейная система неоднородных уравнений с оператором дифференцирования скалярного аргумента. Исследованы задачи Коши для однородных и неоднородных систем уравнений. В соответствии установлены свойства винтовых харктеристик и начальных характеристических интегралов. Разработан новый метод исследования систем с двумя операторами дифференцирования на основе перехода от скалярного аргумента к векторному аргументу. Найдена аналитическая форма единственного решения начальной задачи для системы с двумя операторами дифференцирования в векторно-матричной форме. Получены интегральные представления единственного решения системы уравнений с оператором дифференцирования в векторной форме заданной на цилиндрической поверхности, в случаях тривиального начального условия и периодического начального условия. В результате исследования разработан новый метод исследования начальных задач для линейных однородных и неоднородных систем с двумя операторами дифференцирования на основе перехода к системе с одним оператором дифференцирования, исходя из которого приведена схема исследования таких систем.

В статье были использованы результаты и методы научного проекта «Метод периодических характеристик в исследовании колебаний в системах с оператором дифференцирования по диагонали».

Ключевые слова: оператор дифференцирования, начальные условия, единственность решения, периодичность, винтовые характеристики, линейная система, матрица.