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OSCILLATION CRITERIA OF A SIXTH-ORDER HALF-LINEAR DIFFERENTIAL EQUATION WITH DELAY

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Introduction.

In this work, we consider a sixth-order half-linear differential equation for the p–Laplacian with delay

$$\left(r(t)\left|x^{(V)}(t)\right|^{p-2}x^{(V)}(t)\right)' + q(t)\left|x(\tau(t))\right|^{p-2}x(\tau(t)) = 0, \qquad (1)$$

here $p > 1$, is a real number.

Let the coefficients of the equation satisfy the following conditions:

(C1) $r \in C^1([t_0,\infty),[0,\infty)), r(t) > 0, r'(t) \geq 0,$
$q \in C([t_0,\infty),[0,\infty)),\ q(t) > 0,$

(C2) $\tau(t) \in C([t_0,\infty),R), \tau(t) \leq t,$
$\lim_{t\to\infty} \tau(t) = \infty,$

(C3) $\int_{t_0}^{\infty} \frac{1}{r^{1/(p-1)}(s)} ds = \infty.$

Definition 1 [1]. A nontrivial solution to equation (1) is called oscillatory if for any $T > 0$ it has an infinite set of zeros in (T,∞).

Definition 2 [1]. Equation (1) is called oscillatory if all its solutions are oscillatory.

Linear and nonlinear differential equations (ordinary and partial differential) with delay arise in the mathematical modeling of phenomena and processes in various fields of theoretical physics, mechanics, control theory, biology, biophysics, biochemistry, medicine, ecology, economics and technical applications. Let us present some factors that lead to the need to introduce delay into mathematical models described by differential equations. In biology and biomechanics, the delay is due to the limited speed of transmission of nervous and muscle reactions in living tissues; in medicine – in problems of the spread of infectious diseases – the delay time is determined by the incubation period (the period of time from the moment of infection to the first signs of the disease); in population dynamics, the delay is due to the fact that individuals participate in reproduction only after reaching a certain age; in control theory, delay is usually associated with the finite speed of signal propagation and the limited speed of technological processes; also, equations with delay are often used when describing dynamic processes in the mechanics of a deformable solid body of a medium with hereditary properties, in thermodynamics - when describing irreversible processes, in electrodynamics - when taking into account the finiteness of the interaction rate, in technology - when taking into account the delay in the transfer of energy, materials and signals , in economics - when taking into account the delay time of capital turnover [2-9]. In particular, differential equations with p-Laplacian like operators, as the classical half-linear or Emden–Fowler differential equations, have numerous applications in the study of non-Newtonian fluid theory, porous medium problems, chemotaxis models, etc.; see [10-13].

In 2014, T. Li co-authors in [14] proposed several open problems to study the qualitative properties of solutions of differential equations, and the authors used the Riccati method to find the oscillation conditions of the studied equations.

Using Riccati and comparison methods, O. Basigifan and co-authors in [15] obtained the oscillation criteria of a fourth-order delay half-linear differential equation for a p-Laplacian like operator with different parameters p_1, p_2, and at the end of the article they gave two examples demonstrating the significance of the conclusions.

On the basis of the above discussion, we will establish criteria for the oscillation of equation (1).

Materials and methods of research.

The following lemmas are necessary in the process of proving the fluctuation criteria of equation (1). We quote them without proof.

Lemma 1 [16]. Let $h \in C^n([t_0,\infty),(0,\infty))$. *Suppose that* $h^{(n)}(t)$ *is of a fixed sign on* $[t_0,\infty)$. *Moreover,* $h^{(n)}(t)$ *not identically zero and that there exists* $t_1 \geq t_0$ *such that, for all* $t \geq t_1$,

$$h^{(n-1)}(t)h^{(n)}(t) \leq 0.$$

If *we have* $\lim_{t\to\infty} h(t) \neq 0$, *then there exists* $t_\lambda \geq t_0$ *such that*

$$h(t) \geq \frac{\lambda}{(n-1)!} t^{n-1} |h^{(n-1)}(t)|$$

for every $\lambda \in (0,1)$ and $t \geq t_\lambda$.

Lemma 2 [17]. *If the function* x *satisfies the conditions* $x^{(i)}(t) > 0, i = 0,1,\dots n$, *and* $x^{(n+1)}(t) < 0$, *then*

$$\frac{x(t)}{t^n/n!} \geq \frac{x'(t)}{t^{n-1}/(n-1)!}.$$

Lemma 3. *Let* (C1), (C2), (C3) *hold. If x is an eventually positive solution of* (1), *then for all* $t \geq t_0$ *one of the following conditions is satisfied*

(I) $x > 0, x' > 0, x'' > 0, x''' > 0, x^{(IV)} > 0, x^{(V)} > 0, x^{(VI)} \leq 0$;

(II) $x > 0\ x' < 0, x'' > 0, x''' < 0, x^{(IV)} > 0, x^{(V)} < 0, x^{(VI)} \leq 0$.

Proof. The proof is obvious and therefore is omitted.

Main results.

First we prove the following comparison theorem.

Theorem 1. *Let* (C1), (C2), (C3) *hold. If the linear first order differential equation with delay*

$$\eta'(t)+\frac{\lambda^{p-1}}{120^{p-1}}\frac{q(t)\tau^{5(p-1)}(t)}{r\big(\tau(t)\big)}\eta\big(\tau(t)\big)=0 \tag{2}$$

is oscillatory, then the equation (1) *is also oscillatory.*

Proof.

Let's prove it by contradiction. Let (1) have a nonoscillatory solution in $t\in[t_0,\infty)$. Then, by definition, there exist $t_1\geq t_0$ such that $x(t)\ >\ 0$ and $x(\tau(t))>0$ for $t\geq\ t_1$.

Let us introduce the notation

$$\eta(t):=r(t)\left(x^{(V)}(t)\right)^{p-1}.$$

Putting into equation (1), we get

$$\eta'(t)+q(t)x^{p-1}\big(\tau(t)\big)=0. \tag{3}$$

Since x is positive and increasing, we see $\lim\limits_{t\to\infty}x(t)\neq\ 0$.

According to Lemma 3, two cases are possible.

Consider case (I). Then, based on Lemma 2, the following inequality holds:

$$\frac{x(t)}{t}\geq\frac{x'(t)}{5}. \tag{4}$$

We can derive two sides from this

$$\left(\frac{x(t)}{t}\right)'\geq\frac{x''(t)}{5},$$

or (4) taking into account we find

$$\frac{x(t)}{t^2}\geq\frac{x''(t)}{20}.$$

Next, each time using (4), we sequentially differentiate the resulting inequalities three times, then we find

$$\frac{x(t)}{t^5}\geq\frac{x^{(V)}(t)}{120}.$$

Because the $\tau(t)>0$, then

$$\frac{x\big(\tau(t)\big)}{\tau^5(t)}\geq\frac{x^{(V)}(t)}{120},$$

from here we get that

$$x^{p-1}\big(\tau(t)\big)\geq\frac{\tau^{5(p-1)}(t)}{120^{p-1}}\left(x^{(V)}\big(\tau(t)\big)\right)^{p-1}. \tag{5}$$

In (II) case, according to Lemma 1, the following inequality holds for all $\lambda\ \in(0,1)$:

$$x^{p-1}\big(\tau(t)\big) \geq \frac{\lambda^{p-1}}{120^{p-1}} \tau^{5(p-1)}(t) \left(x^{(V)}\big(\tau(t)\big)\right)^{p-1}. \qquad (6)$$

So, we compare (5) and (6) and make sure that (6) is valid in both cases.

Next, applying inequality (6) to (3), we get

$$\eta'(t) + \frac{\lambda^{p-1}}{120^{p-1}} q(t)\tau^{5(p-1)}(t) \left(x^{(V)}\big(\tau(t)\big)\right)^{p-1} \leq 0.$$

$\eta(t)$ taking into account the notation of the function, we find

$$\eta'(t) + \frac{\lambda^{p-1}}{120^{p-1}} \frac{q(t)\tau^{5(p-1)}(t)}{r\big(\tau(t)\big)} \eta\big(\tau(t)\big) \leq 0.$$

By using the Theorem 1 in [18], we find that the equation (2) also has a positive solution. This leads to a contradiction to the statement of the theorem. That is, if equation (2) is oscillating, then equation (1) is also oscillating. So, the theorem is fully proved.□

Theorem 1. *Let* (C1), (C2), (C3) *hold. If*

$$\liminf_{t\to\infty} \int_{\tau(t)}^{t} \frac{\lambda^{p-1}}{120^{p-1}} \frac{q(s)\tau^{5(p-1)}(s)}{r\big(\tau(s)\big)} ds > \frac{1}{e}, \ \ \lambda \in (0,1), \qquad (7)$$

then the equation (1) *is oscillatory.*

Proof.

Based on Theorem 1, equation (1) is oscillatory if equation (2) is oscillatory. And equation (2) is a first-order linear differential equation with delay τ(t). Then, according to Theorem 1 in [19], equation (2) is oscillatory if condition (7) is fulfilled. The proof is complete.□

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АЛТЫНШЫ РЕТТІ КЕШІГУІ БАР ЖАРТЫЛЫЙ СЫЗЫҚТЫ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУДІҢ ТЕРБЕЛІС КРИТЕРИІ

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Андатпа. Бұл жұмыста біз $\left(r(t)\left|x^{(V)}(t)\right|^{p-2}x^{(V)}(t)\right)' + q(t)\left|x\left(\tau(t)\right)\right|^{p-2}x\left(\tau(t)\right) = 0$ түріндегі кешігетін аргументі бар p-Лапласиан типті оператор үшін алтыншы ретті жартылай сызықты дифференциалдық теңдеу қарастырамыз. Мұндағы теңдеуге кіретін айнымалы коэффициенттері берілген шарттарды қанағаттандырады. Кешігуі бар сызықты және сызықты емес дифференциалдық теңдеулер (жай және дербес туындылы теңдеулер) теориялық физиканың, механиканың, басқару теориясының, биологияның, биофизиканың, биохимияның, медицинаның, экологияның, экономиканың және техникалық қолданудың әртүрлі салаларындағы құбылыстар мен процестерді математикалық модельдеуде пайда болады. Математикалық модельдер мен дифференциалдық теңдеулерде кешігуі болуы, әдетте, алынған шешімдердің орнықтылық облысының тарылуына әкелетін күрделендіретін фактор болып табылады. Кешігуі бар қарапайым дифференциалдық теңдеулерді зерттеу және шешу күрделілігі бойынша кешігуі жоқ дербес туындылы дифференциалдық теңдеулерді зерттеу және шешумен салыстырмалы. Қазіргі уақытта кешігуі бар дифференциалдық теңдеулердің шешімдерінің әртүрлі қасиеттерін зерттеуге арналған көптеген жұмыстар бар. Бұл жұмыстың мақсаты – қарастырылып отырған дифференциалдық теңдеудің тербелмелімділігін зерттеу болып табылады. Тербелмелі критерийді алу үшін Риккати әдісі қолданылады және кешігуі бар бірінші ретті дифференциалдық теңдеумен салыстыру теоремасы дәлелденеді, ал оған бұрын белгілі тербеліс критерийін қолдануға болады.

Түйін сөздер: тербелімділігі, жартылай сызықты дифференциалдық теңдеулер, p-лапласиан, алтыншы ретті, кешігуі бар дифференциалдық теңдеулер, салыстыру теоремасы

КРИТЕРИИ ОСЦИЛЛЯТОРНОСТИ ПОЛУЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ШЕСТОГО ПОРЯДКА С ЗАПАЗДЫВАНИЕМ

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Аннотация. В данной работе мы рассматриваем полулинейное дифференциальное уравнение шестого порядка для оператора типа p-Лапласиана с запаздывающим аргументом вида $\left(r(t)\left|x^{(V)}(t)\right|^{p-2}x^{(V)}(t)\right)' + q(t)\left|x\left(\tau(t)\right)\right|^{p-2}x\left(\tau(t)\right) = 0$. Здесь коэффициенты уравнения удовлетворяют заданным условиям. Линейные и нелинейные дифференциальные уравнения (обыкновенные и в частных производных) с запаздыванием возникают при математическом моделировании явлений и процессов в различных областях теоретической физики, механики, теории управления, биологии, биофизики, биохимии, медицины, экологии, экономики и технических приложениях. Наличие запаздывания в математических моделях и дифференциальных уравнениях является осложняющим фактором, который, как правило, приводит к сужению области устойчивости получаемых решений. Исследование и решение обыкновенных дифференциальных уравнений с запаздыванием по сложности сопоставимы с исследованием и решением уравнений в частных производных без запаздывания. В настоящее время имеется множество работ по изучению различных свойств решений дифференциальных уравнений с запаздыванием. Целью данной работы является изучение осцилляторности рассматриваемого дифференциального уравнения. Для получения критерия осцилляторности используется метод Риккати и доказывается теорема сравнения с дифференциальным уравнением первого порядка с запаздыванием, к которому можно применить известный ранее критерий осцилляторности.

Ключевые слова: осцилляторность, полулинейные дифференциальные уравнения, p-лапласиан, шестой порядок, дифференциальные уравнения с запаздыванием, теорема сравнения