ФИЗИКА-МАТЕМАТИКА ҒЫЛЫМДАРЫ

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THE CALCULATION OF THE RESISTANCE BETWEEN THE DIAGONAL POINTS OF THE TWO-DIMENSIONAL INFINITE GRID WITH SQUARE CELLS

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Аннотация. Мақалада шексіз торлы резистор жүйелерінің әртүрлі түйіндері арасындағы кедергіні есептеу мәселесі қарастырылған. Ертеректе симметрия және суперпозиция әдісімен алынған нәтижелердің жуықтау болатынын растайтын жаңа қарсы мысалдар келтірілген. Шексіз торлы жүйелердің көршілес түйіндері арасындағы кедергіні есептеу үшін эквивалентті кедергі әдісін қолдануының әдептілігі дәлелденді. Бірдей r резисторлардан тұратын шексіз тордың квадрат ұяшықтарының диагональ нүктелері арасындағы кедергі есептеледі. Бұл кедергінің мәні $r/\sqrt{2}$ бұрын симметрия және суперпозиция әдісімен табылған $2r/\pi$ мәннен өзгеше болып шықты.

Түйінді сөздер: кедергіні есептеу; кедергілердіің шексіз екі өлшемді торлары; эквивалентті кедергі әдісі. бірдей

Аннотация. В работе рассмотрена проблема расчета сопротивления между различными узлами бесконечных сеточных резисторных систем. Приведены новые контрпримеры, доказывающие приближенность результатов, полученных ранее методом суперпозиции и симметрии. Доказывается корректность использования метода эквивалентного сопротивления для расчета сопротивления между близлежащими узлами бесконечных сеточных систем. Рассчитано сопротивление между диагональными точками бесконечной сетки одинаковых резисторов r с квадратными ячейками. Для величины этого сопротивления найдено значение $r/\sqrt{2}$, которое отличается от значения $2r/\pi$, полученного ранее методом суперпозиции и симметрии.

Ключевые слова: расчет сопротивления; бесконечные двумерные сетки сопротивлений; метод эквивалентного сопротивления.

Abstract. The paper considers the problem of calculating the resistance between different nodes of infinite grid resistor systems. New counterexamples are presented proving the proximity of the results obtained earlier by the superposition & symmetry method. The correctness of using the equivalent resistance method to calculate the resistance between close nodes of infinite grid systems is proved. The resistance between the diagonal points of an infinite grid of identical resistors r with square cells is calculated. For the value of this resistance, a value is found $r/\sqrt{2}$ that differs from the value $2r/\pi$ obtained previously by the superposition and symmetry method.

Keywords: calculation of resistance; infinite two-dimensional grid of resistances; equivalent resistance method.

1. Introduction. The calculation of the resistance of complex resistor compounds has always attracted the attention of physicists. Many different original methods for calculating endless resistance circuits have been developed [1–4]. The tasks of finding the resistance of infinite grid systems also been included in Irodov's book [5] and were considered in our work [6]. The problems of calculating the resistances of infinite resistor grids, using graphene [7] and thin films [8] in connection with the development of nanotechnologies have become especially urgent.

Let us dwell in more detail on one problem reviewed in the works [2–6]. «There is a boundless wire grid with square cells (Fig. 1). The resistance of each conductor between neighboring grid nodes is *r*. Find the resistance R_{AB} of this grid between two adjacent grid nodes *A* and *B*».

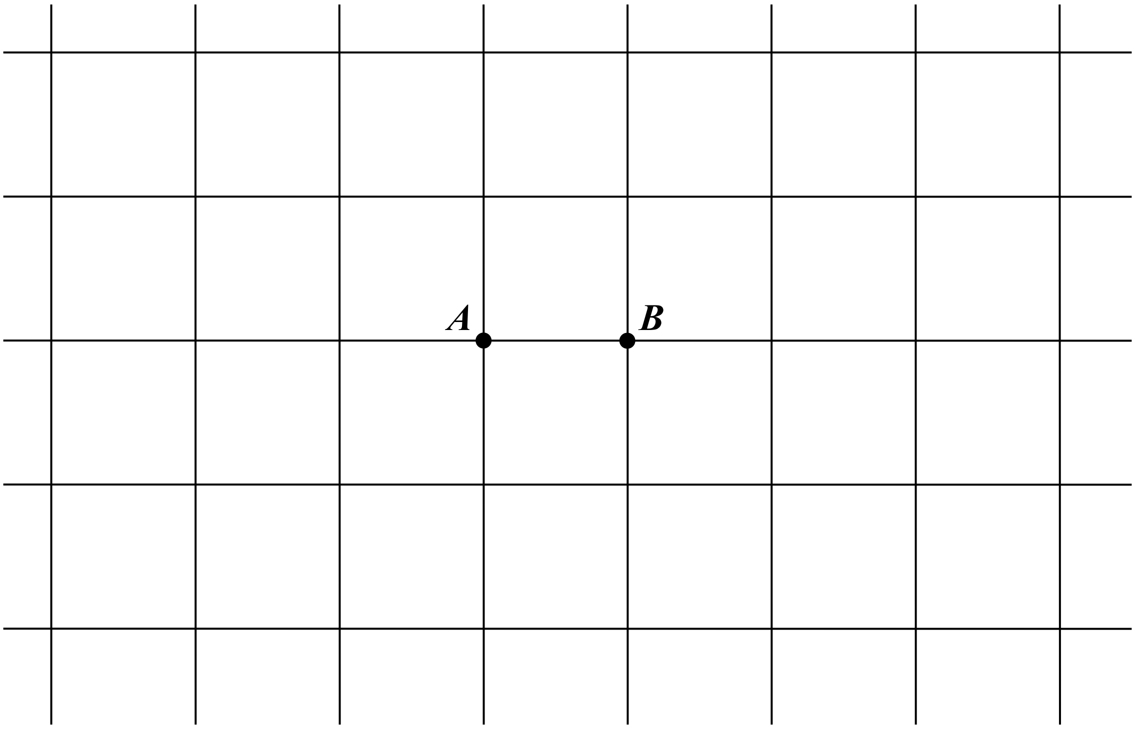

Figure 1. Infinite wire grid with square cells.

This task first appeared in Irodov's booklet of 1979 yeas, where the method of superposition & symmetry was applied and the result was obtained:

$$R_{AB}=\frac{r}{2}. \qquad (1)$$

The same result (1) was also obtained in the works [2–4]. This opinion was especially strengthened in connection with work [4]. In this work, complex mathematics made it possible to obtain a general formula for the resistance between any points of the grid. However, here the method symmetry & superposition is also based, therefore, for the resistance between the nearest points of grid, this formula gives the same result (1).

Now this is a general misconception, which is very difficult to overcome. Of course, the result $r/2$ for resistances fascinates with its simplicity, but it is approximate. Moreover, it gives only a lower bound for estimating the magnitude of the resistance.

In Fig. 2 shows three cases of connecting voltage to the points of the grid. In case *a*) the voltage $+U/2$ is supplied only to point *A*, and in case *b*) the voltage $-U/2$ is supplied only to point *B*. In these cases, the current distribution is symmetrical and they differ only in the direction of the currents.

In case c) voltage $+U/2$ is supplied to point *A*, and voltage $-U/2$ to point *B*. Here *A* and *B* are the nearest points of the grid, between which the resistance r is located. Current $i' \neq i$ approaches point *A*, and the same current i' emerges from point *B*. The current distribution at points *A* and *B* is shown, which corresponds to the symmetry of the problem. A current i_{AB} flows from *A* to *B*. Then according to Ohm's law:

$$U = i' R_{AB} = i_{AB} r \, . \quad (2)$$

Here R_{AB} is the desired grid resistance between the points *A* and *B*. If we make two assumptions that $i' = i$ and $i_{AB} = i/2$, then from formula (2) we get result (1):

$$R_{AB} = \frac{r}{2} \, .$$

In this case, according to the Kirchhoff rule:

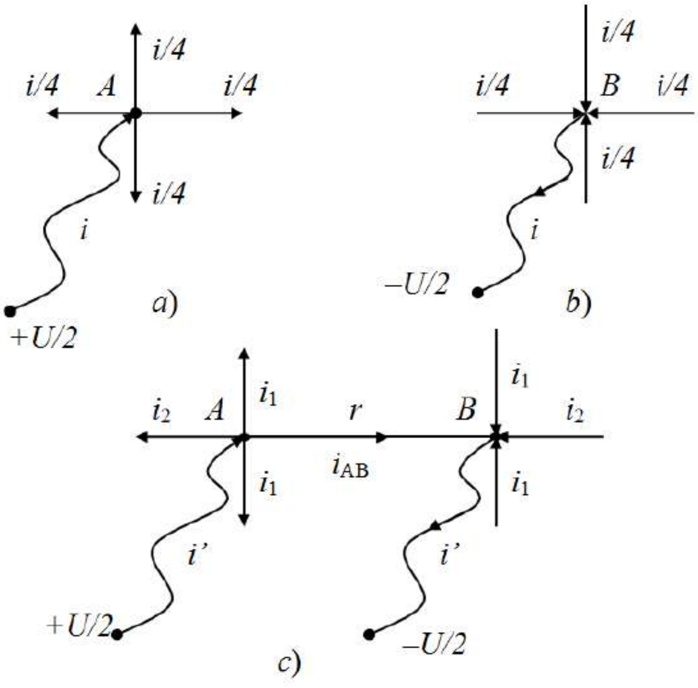

Figure 2. *a*) The potential $+U/2$ only at point *A* (zero at infinity);

b) The potential $-U/2$ only at point *B* (zero at infinity);

c) The potential $+U/2$ at point *A* and the potential $-U/2$ at point *B*.

If $i_{AB} = i/2$, $i_1 = i/4$, then $i_2 = 0$. So the distribution of currents at points A and B has changed, weird – $i_2 = 0$.

Symmetry in case c) has changed. Note that a line with a potential equal to zero runs in the middle between the points A and B and goes to infinity. Since the potential difference between points *A* and B and infinity is not equal to zero, the currents $i_2 \neq 0$, in addition, $i' = i$ and $i_{AB} > i'/2$.

These three factors indicate that the result $R_{AB} = r/2$ is an approximation, more precisely even $R_{AB} > r/2$. In our work [6] we used the equivalent resistance method and obtained the following result:

$$R_{AB} = \frac{2\left(\sqrt{2}-1\right)+\sqrt{2\sqrt{2}-1}}{2\sqrt{2}+\sqrt{2\sqrt{2}-1}} r \cong 0.521602\, r, \qquad (3)$$

and the following values of the currents: $i_{AB} \cong 0.522\, i'$, $i_1 \cong 0.207\, i'$, $i_2 \cong 0.064\, i'$. It is easy to verify that in this case:

$$i' = i_{AB} + 2i_1 + i_2.$$

So, here everything is in order with the Kirchhoff rule in the points A and B.

But the result, which we obtained $R_{AB} \cong 0.522\, r$, is just only slightly superior to the result $r/2$, which confirms its correctness. The arguments we have presented, I think, prove that the result $r/2$ is only approximate.

2. Calculation of the resistance between the diagonal points of the infinite grid

Here we give a solution to the problem of an infinite grid with square cells by the same resistances *r*, as shown in Fig. 3. Suppose it is necessary to find the resistance R_{AC} of the lattice between the two points *A* and *C*. The solution of the problem by the method of superposition & symmetry in [4] leads to the following result:

$$R_{AC} = \frac{2r}{\pi} \cong 0.636620\, r. \qquad (4)$$

Figure 3 shows a part of the grid, and the dashed lines show the directions along which it is necessary to make cuts. First, we cut along the rays issuing from the nodes *A* and *C*, thus breaking the entire grid into two half-planes with the same resistances *R*. As a result, we obtain for the resistance formula:

$$R_{AC} = \frac{R}{2}. \qquad (5)$$

Now you need to find the half-plane resistance R. To do this, we draw a second section from the points A_1 and C_1, as shown in Fig. 3.

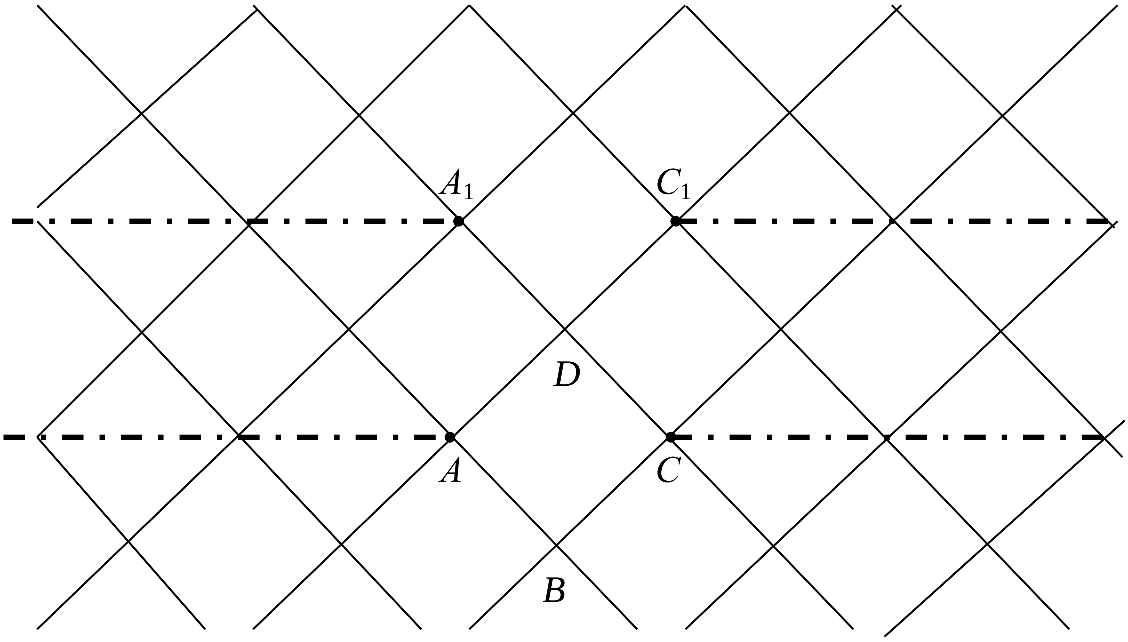

Figure 3. Infinite grid with square cells and cutting line directions

To determine R, we construct the following equivalent circuit shown in Fig. 4.

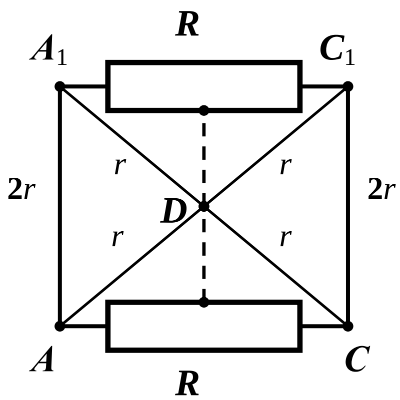

Figure 4. Equivalent circuit for calculation of the half-plane resistance R

In Fig. 4 shows a dashed vertical line passing through the point D and dividing the circuit into two symmetrical parts. Connecting the point D with the middle of the resistance R, we calculate the resistance of the resulting compound. First, find the resistance of the parallel connection $\frac{R}{2}$ and r get:

$$r' = \frac{\frac{R}{2} r}{\frac{R}{2} + r} = \frac{R r}{R + 2r} \quad (6)$$

Then, connecting in series r' with $2r$, we find:

$$r'' = \frac{R r}{R + 2r} + 2r = \frac{3R r + 4r^2}{R + 2r} \quad (7)$$

Now we make up the equation to determine the half-plane resistance R:

$$\frac{R}{2} = \frac{r'' r}{r'' + r} \quad (8)$$

Substituting (7) into (8), we obtain:

$$\frac{R}{2} = \frac{3Rr^2 + 4r^3}{4Rr + 6r^2} \quad (9)$$

Here will we find

$$R = \sqrt{2}\, r \quad (10)$$

Substituting (10) into (5), we obtain the desired resistance between the diagonal points:

$$R_{AC} = \frac{R}{2} = \frac{r}{\sqrt{2}} \cong 0.707107\, r \quad (11)$$

It is easy to verify that the difference with the result obtained by the method of superposition & symmetry (4) is 10%.

3. Conclusion. Thus, in work [6] and this article we have developed a method for calculating infinite resistance grids, which allows us to find the exact resistance between the nodes of such grids. It is shown that the calculation method used in [2–5], based on the principles of superposition & symmetry, gives only an approximate underestimated result for this resistance. As we have shown in our works for the grid with square cells, the difference in results does not exceed 10%, and for the grid with triangular elements it approaches 15%.

The method of calculating the resistances of the infinite grids, which uses the principles of symmetry & superposition, is quite good, and its simplicity makes it very attractive for an approximate evaluation of the resistances of various infinite configurations of resistances. So, for example, for resistance R_{AB} between two nearest points of an infinite grid with hexagonal cells (a configuration of graphen) we get value $R_{AB} = 2r/3$, and for a volume the infinite 3D grid with cubic cells – $R_{AB} = r/3$.

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ЭЛЕКТРОТЕХНИКА НЕГІЗІ ПӘНІН КУРСАНТТАРДЫҢ МЕҢГЕРУІНДЕ ТІРЕК – ЛОГИКАЛЫҚ КОНСПЕКТ ТЕХНОЛОГИЯСЫНЫҢ МАҢЫЗЫ

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Аңдатпа. Бұл мақалада Қазақстанда жоғары оқу орындарында жаңа педагогикалық оқыту технологиясы негізінде жүріп жатқан ғылыми жұмыстарға қысқаша шолу жасап, электротехника пәнін меңгеруде қолданылатын оқыту әдістемесі мен педагогикалық технологиясының ерекшіліктеріне талдау жасадым. Қортындылай келе Әскери оқу орынында электротехника негізі пәнін оқытуда да өзіндік ерекшеліктерін ескере отырып , оқу процесінде В.Ф.Шаталовтың негізінде жасалған тірек конспект оқыту технологиясын қолдану тиімді деп шештім.

Түйін сөздер: жаңа педагогикалық оқыту технологиясына шолу, электротехника пәнін меңгеруде қолданылатын оқыту әдістемесі мен педагогикалық технологиясының ерекшіліктеріне талдау, әскери педагогикалық технологиядағы басты міндет, электротехника негізі пәнін оқытуда тірек конспект оқыту технолгиясын қолдану.

Аннотация. В данной статье представлен краткий обзор научных работ, проводимых в Казахстане на основе новой технологии педагогического обучения, анализ особенностей методики преподавания и