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# ON A NUMERICAL METHOD FOR SOLVNIG BOUNDARY VALUE PROBLEM FOR A LOADED HYPERBOLIC EQUATION

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**Annotation.** In this communication, we consider a semi-periodic boundary value problem for a loaded linear hyperbolic equation. To find a numerical solution, the problem is approximated by the finite difference method using a semi-implicit scheme. The sweep method is used to solve the difference problem.

**Keywords:** laded hyperbolic equation, boundary value problem, numerical method, semi implicit scheme, finite difference method, sweep method.

We consider the semi-periodic boundary value problems for the linear loaded hyperbolic equation

$$\frac{\partial^2 u(x,t)}{\partial t \partial x} = A(x,t)\frac{\partial u(x,t)}{\partial x} + B(x,t)\frac{\partial u(x,t)}{\partial t} + C(x,t)u(x,t) + f(x,t) + A_0(x,t)\frac{\partial (x_0,t)}{\partial x}, (x,t) \in [0,2] \times [0,2\pi] \quad (1)$$

subject to the initial condition

$$u(0,t) = \varphi(t), \quad t \in [0,2\pi], \quad (2)$$

and boundary condition

$$u(x,0) = u(x,2\pi) = 0, \quad x \in [0,2], \quad (3)$$

where $A(x,t), B(x,t), C(x,t)$ and $f(x,t)$ are continuous on $\Omega$, $\varphi(t)$ is continuous differentiable on $[0,2]$ and $x_0$ is loaded point.

The development of a numerical technique for a hyperbolic equation combining classical, integral boundary conditions and nonlocal conditions has been studied by many authors [1-5]. In [6], a new numerical method is presented, which is used to solve the wave equation with nonlocal boundary conditions. In many works, initial-boundary value problems are considered for hyperbolic, loaded - hyperbolic equations of the second and third orders, but the equations do not contain a mixed derivative. In [7], equations with a mixed derivative of the third order are considered and the solution of the Goursat problem for a loaded hyperbolic equation of the second order, proposed as a mathematical model of the Aller equation under certain conditions, is written out in an explicit form.

For numerical simulations, we introduce a space time grid with steps h, τ respectively, in the variables x, t:

$$\omega_{h,\tau} = \{x_i = ih, i = \overline{0,N}; t_k = k\tau, k = \overline{0,M};\} \qquad (4)$$

Where $h = \frac{l}{N}, \tau = \frac{T}{M}$.

On this grid we approximate the problem (1)–(3) using the finite difference method.

Consider the semi implicit scheme for the problem (1)–(3).

$$\frac{u_{i+1}^{k+1} - u_{i+1}^{k} - u_{i-1}^{k+1} - u_{i-1}^{k}}{2h\tau} = A_i^k \frac{u_{i+1}^{k+1} - u_{i-1}^{k+1}}{2h} + B_i^k \frac{u_i^{k+1} - u_i^k}{\tau} + C_i^k u_i^k + f_i^k + A_{0}{}_i^k \frac{u_{i+1}^k - u_{i-1}^k}{2h} \qquad (5)$$

For $(i, k) \in \omega_{h,\tau}$, with initial condition

$$u_0^k = \varphi_k \qquad (6)$$

and with boundary conditions

$$u_i^0 = 0, u_N^0 = 0 \qquad (7)$$

It is well-known, that the implicit scheme and it has accuracy order $O(\tau + |h|^2)$.

Equation (5) can be reduced to the most general form:

$$a_i u_{i+1}^{k+1} - b_i u_i^{k+1} + c_i u_{i-1}^{k+1} = F_i \qquad (8)$$

where

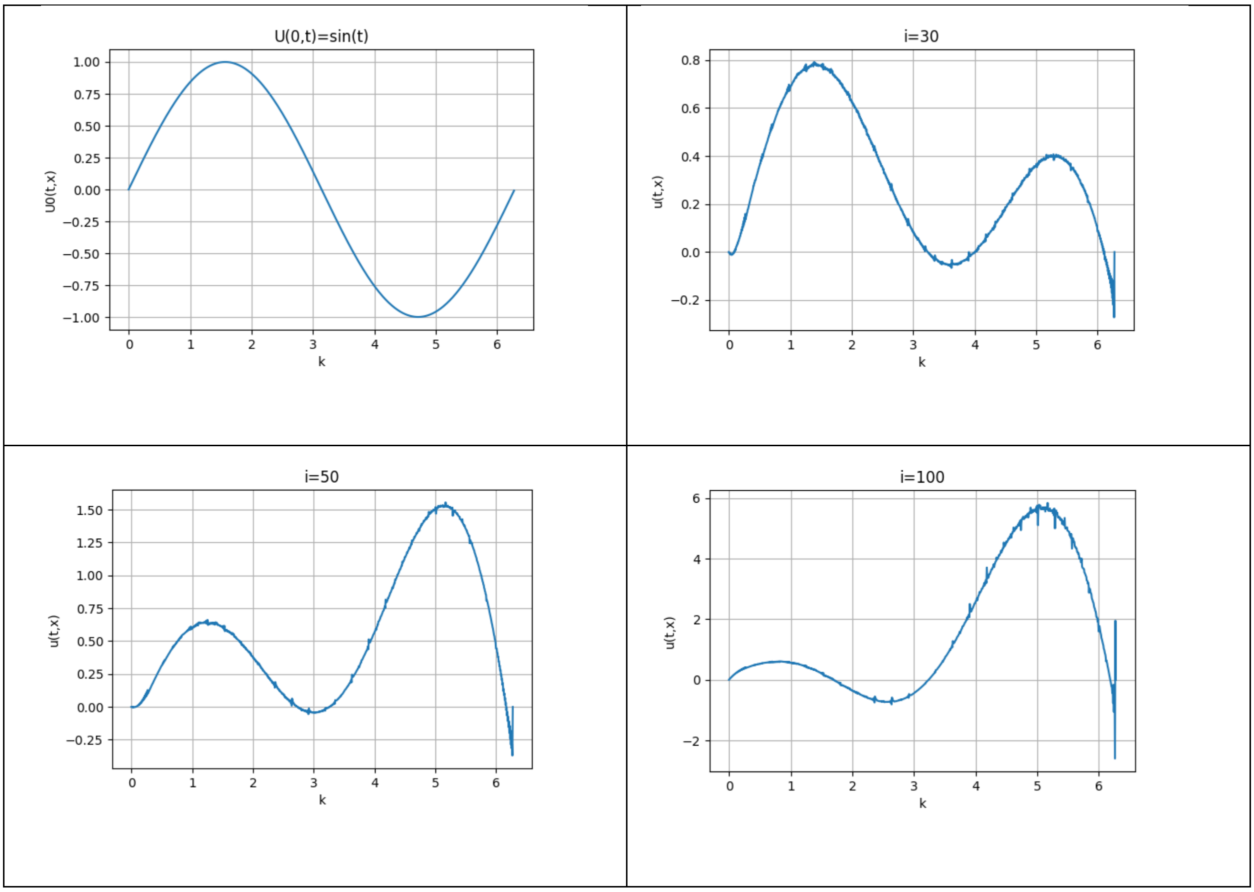
$$a_i = 1 - \tau A_i^k; b_i = 2hB_i^k + 2h\tau C_i^k;\ c_i = \tau A_i^k - 1;$$

$$F_i = \tau A_i^k \left(u_{i+1}^k - u_{i-1}^k\right) - 2\mathrm{h}B_i^k u_i^k + 2h\tau f_i^k + \tau A_{0}{}_i^k (u_{i+1}^k - u_{i-1}^k)$$

Such equations are called three-point difference equations of the second order. System (8) has a tridiagonal structure and Equation (8) is linear equation systems of the form [A]U = f as shown in Equation (9).

$$\begin{bmatrix} b_1 & a_1 & 0 & \cdots & \cdots & \cdots & 0 \\ c_1 & b_2 & \ddots & \ddots & & & \vdots \\ 0 & c_2 & \ddots & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & a_{n-2} & 0 \\ \vdots & & & \ddots & & b_{n-1} & a_{n-1} \\ 0 & \cdots & \cdots & \cdots & 0 & c_{n-1} & b_n \end{bmatrix} \times \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ u_n \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ f_n \end{pmatrix} \qquad (9)$$

Because a non-stationary problem is considered, system (8) must be solved at each time step. We solve the difference equation (8) by sweep method [8].

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## ЖҮКТЕЛГЕН ГИПЕРБОЛАЛЫҚ ТЕҢДЕУ ҮШІН ШЕТТІК ЕСЕПТІ ШЕШУДІҢ САНДЫҚ ӘДІСІ ТУРАЛЫ

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**Андатпа.** Бұл хабарламада жүктелген сызықтық гиперболалық теңдеу үшін жартылай периодты шеттік есеп қарастырылады. Есептің сандық шешімін табу үшін жартылай айқын схема қолданылады, ақырлы айырымдар әдісі көмегімен есеп жуық есепке келтіріледі. Қуалау әдісі арқылы айырымдық есеп шығарылады.

**Түйінді сөздер:** жүктелген гиперболалық теңдеу, шеттік есеп, сандық әдіс, жартылай айқын схема, ақырлы айырымдар әдісі, қуалау әдісі.

## О ЧИСЛЕННОМ МЕТОДЕ РЕШЕНИЯ КРАЕВОЙ ЗАДАЧИ ДЛЯ НАГРУЖЕННОГО ГИПЕРБОЛИЧЕСКОГО УРАВНЕНИЯ

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**Аннотация.** В этом сообщении рассматривается полупериодическая краевая задача для нагруженного линейного гиперболического уравнения. С использованием полуявной схемы находятся численное решение задачи, задача аппроксимируется методом конечных разностей. Метод прогонки используется для решения разностной задачи.

**Ключевые слова:** нагруженное гиперболическое уравнение, краевая задача, численный метод, полуявная схема, метод конечных разностей, метод прогонки.