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INVERSE CAUCHY PROBLEMS FOR POLYHARMONIC HEAT EQUATIONS

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Annotation. We recover a time-dependent coefficient in inverse problems for polyharmonic heat equations. This method can be applied to many evolution equations.

Keywords: polyharmonic heat equations, inverse Cauchy problems, fundamental solutions.

We consider the following inverse Cauchy problems

$$\partial_t u_1(x,t) + a(t)(-\Delta_x)^m u_1(x,t) = 0, \quad x \in R^n,\ 0 < t \le T, \tag{1}$$

$$u_1(x,0) = \Phi(x), \quad x \in R^n, \tag{2}$$

$$u_1(q,t) = h_1(t), \qquad 0 \le t \le T, \tag{3}$$

and

$$\partial_t u_2(x,t) + a(t)(-\Delta_x)^m u_2(x,t) = 0, \quad x \in R^n,\ 0 < t \le T, \tag{4}$$

$$u_2(x,0) = -(-\Delta_x)^m \Phi(x), \quad x \in R^n, \tag{5}$$

$$u_2(q,t) = h_2(t), \qquad 0 \le t \le T, \tag{6}$$

where $a(t)$ is unknown, $\Omega \subseteq R^n$, $q \in \Omega$ is an arbitrarily fixed point and

First, we show existence and uniqueness of the Cauchy problem (1)-(2). If $a(t)$ is continuous, then the equation (1) is parabolic in the sense of Petrovskii [1, 2].

Theorem 1 ([1]) *Let* $a(t)$ *be* continuous *on* $[0,T]$. *Let* $\Phi \in C^{2m,\gamma}, 0 < \gamma < 1$ *be defined by*

$$\Phi(x) = \begin{cases} \varphi(x), & x \in \Omega \\ 0, & x \notin \Omega \end{cases} \tag{7}$$

Then, the Cauchy problem (1) – (2) has the following unique solution

$$u(x,t)=\int_{R^n} E_a(x-y,t)\Phi(y)dy=\int_{\Omega} E_a(x-y,t)\varphi(y)dy$$

and it belongs to $C^{2m,\gamma,0}(\Omega\times[0,T])$, *where the fundamental solution of (1) is given by*

$$E_a(x,t):=(2\pi)^{-n}\int_{R^n} e^{ix\cdot s-|s|^{2m}a_1(t)}ds\,.$$

Also, it can be reduced to the one-dimensional integral

$$E_a(x,t)=(2\pi)^{\frac{n}{2}}(a_1(t))^{-\frac{n}{2m}}\int_0^{\infty} e^{-r^{2m}}r^{\frac{n}{2}}J_{\frac{n-2}{2}}\left(r\,|\,x\,|\,a_1(t)^{-\frac{1}{2m}}\right)dr,$$

where, J_k is the Bessel function of the first kind (see, [2, 183-184 pp.]). Now we present our main result.

Theorem 2 ([3]) *Let* $\Phi(x)$ *be a function of* $C^{4m,\gamma}$. *Let the additional data* h_1 *and* h_2 *satisfy the assumptions:*

i. $h_1\in C^1[0,T]$;

ii. $h_2\in C[0,T]$ *such that* $h_2(t)\neq 0$ *for all* $0\le t\le T$ *(which also implies* $h_2(0)=u_2(x,0)|_{x=q}=-(-\Delta_y)^m\Phi(x)|_{x=q}\neq 0$*);*

iii. $\dfrac{h_1'(t)}{h_2(t)}$ *ensures that the equation (1) is uniformly parabolic in the sense of Petrovskii.*

Then the inverse problem (1)-(6) has a unique solution and the coefficient $a(t)$ *is given explicitly*

$$a(t)=\frac{h_1'(t)}{h_2(t)},\qquad 0\le t\le T.$$

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ПОЛИГАРМОНИЯЛЫҚ ЖЫЛУ ТЕҢДЕУЛЕРІНЕ АРНАЛҒАН КОШИДІҢ КЕРІ ЕСЕПТЕРІ

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Аңдатпа. Полигармониялық жылу теңдеулері үшін кері есептердегі уақытқа тәуелді коэффициентті қалпына келтіріледі. Бұл әдісті көптеген эволюциялық теңдеулерге қолдануға болады.

Түйінді сөздер: жылуөткізгіштіктің полигармоникалық теңдеулері, Кошидің кері есептері, іргелі шешімдер.

ОБРАТНЫЕ ЗАДАЧИ КОШИ ДЛЯ ПОЛИГАРМОНИЧЕСКИХ УРАВНЕНИЙ ТЕПЛОПРОВОДНОСТИ

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Аннотация. Восстанавливливается зависящий от времени коэффициент в обратных задачах для полигармонических уравнений теплопроводности. Этот метод может быть применен ко многим эволюционным уравнениям.

Ключевые слова: полигармонические уравнения теплопроводности, обратные задачи Коши, фундаментальные решения.