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INTEGRO-DIFFERENTIAL EQUATIONS WITH BOUNDARY CONDITIONS AS A BIOLOGICAL MODEL

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Annotation. A boundary value problem for integro-differential equation is considered. The method proposed by D.S. Dzhumabaev for solving this problem is described and its results are commented. A new general solution of integro-differential equations is constructed and its properties are established. Algorithms for finding solutions to the boundary value problems for integro-differential equations are constructed and conditions for unique solvability are established in the terms of initial data. The results are illustrated by examples.

Keywords: integro-differential equations, boundary value problems, new general solution, algorithm, solvability.

On the interval $[0,T]$ consider the following boundary value problem for system of integro-differential equations

$$\frac{dx}{dt} = A(t)x + \int_0^T K(t,s)x(s)ds + f(t), \qquad t \in (0,T), \quad x \in R^n, \tag{1}$$

$$Bx(0) + Cx(T) = d, \qquad d \in R^n, \tag{2}$$

where $x(t)$ is unknown vector function, the $(n \times n)$ matrix $A(t)$, the n vector function $f(t)$ are continuous on $[0,T]$, the $(n \times n)$ matrix $K(t,s)$ is continuous on $[0,T] \times [0,T]$, the B, C are constant $(n \times n)$ matrices.

The solution of problem (1), (2) is defined as a function $x(t) \in C([0,T], R^n)$ continuously differentiable on $(0,T)$ and satisfying the integro-differential equation (1) and the boundary condition (2).

The main methods used for the investigation and solution of the boundary-value problem (1), (2) are the Nekrasov method [1] and the method of Green functions [2]. The application of these methods requires the unique solvability of certain auxiliary problems.

In the Nekrasov method, it is assumed that the Fredholm integral equation of the second kind

$$x(t)=\int_0^T M(t,s)x(s)ds+F(t), \qquad t\in[0,T],$$

with the kernel $M(t,s)=\int_s^T K(t,\tau)X(\tau)d\tau\, X^{-1}(s)$, where $X(t)$ is the fundamental matrix of the differential part of IDE (1) and $F(t)\in C([0,T],R^n)$, is uniquely solvable.

If the latter one is uniquely solvable, then the general solution to Fredholm integro-differential equation (IDE) can be written down via the resolvent of integral equation. The solvability of problem (1), (2) is equivalent to the solvability of linear system of algebraic equations compiled by the general solution and condition (2).

The method of Green functions can be used for the solution of problem (1), (2) under the assumption of unique solvability of the boundary-value problem for the differential part of the IDE (1), i.e., according to this method, problem (1), (2) with $K(t,s)=0$ must be uniquely solvable.

In this method, the integral term also refers to the right-hand side of differential equation. Under assumption on unique solvability of boundary value problem for the differential equation we construct its Green's function and then reduce the origin boundary value problem for Fredholm integro-differential equation (1), (2) to the Fredholm integral equation of second kind. Solving this equation, we find the desired function.

Since the unique solvability of auxiliary problems is not a necessary condition for the existence of the solution of the original boundary value problem, the Nekrasov method and the method of Green functions do not enable us to establish necessary and sufficient conditions for the solvability of problem (1), (2) [3].

In the present communication we are described the Dzhumabaev parametrization method [4] for finding of solutions to the boundary value problem for Fredholm integro-differential equation.

Given a step $h>0: Nh=T$, we introduce the partition $[0,T)=\bigcup_{r=1}^{N}[(r-1)h,rh)$.

Necessary and sufficient conditions for solvability, including the unique solvability of problem (1), (2) were obtained in terms of a matrix $Q_{*,*}(h)$ constructed from the fundamental matrix of the differential part of system (1), the matrices in boundary conditions (2), and the resolvent of an auxiliary Fredholm integral equation of the second kind [5].

Thus, if the parametrization method is applied to problem (1), (2), we also have to solve an intermediate problem, namely, the special Cauchy problem for integro-differential equations or the equivalent system of integral equations [5]. However, in contrast to the above methods, the partition step $h>0:Nh=T$ can always be chosen so that special Cauchy problem for integro-differential equations is uniquely solvable.

Conditions for the convergence of algorithms of the Dzhumabaev parametrization method for finding of solutions to the boundary value problems for Fredholm integro-differential equation were determined [6-9].

A concept of new general solution to Fredholm integro-differential equations were introduced and its properties were established [10]. Criterions for the unique solvability of the boundary value problem for Fredholm integro-differential equation were obtained in the terms of the initial data and new general solution [10-12]. This approach are extended to the boundary value problems for loaded differential equations [13], for nonlinear integro-differential equations [14]. A modification this method for solving to the boundary value problem for Fredholm integro-differential equation and some numerical approximation its solution are offered [15-16]. Finally, we are also extended Dzhumabaev parametrization method to solve problem with parameter for Fredholm integro-differential equations [17-20].

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ШЕТТІК ШАРТТАРЫ БАР ИНТЕГРАЛДЫҚ-ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР БИОЛОГИЯЛЫҚ МОДЕЛЬ РЕТІНДЕ

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Аңдатпа. Интегралдық-дифференциалдық теңдеу үшін шеттік есеп қарастырылады. Осы есепті шешуге Д.С.Жұмабаев ұсынған әдіс сипатталады және оның нәтижелеріне түсініктеме беріледі. Интегралдық-дифференциалдық теңдеулердің жаңа жалпы шешімі тұрғызылады және оның қасиеттері орнатылады. Интегралдық-дифференциалдық теңдеулер үшін шеттік есептердің шешімін табуға арналған алгоритмдер құрылады және бірмәнді шешілімділігінің шарттары бастапқы берілімдер терминінде орнатылады. Нәтижелер мысалдармен көрнекіленеді.

Кілттік сөздер: интегралдық-дифференциалдық теңдеулер, шеттік есептер, жаңа жалпы шешім, алгоритм, шешілімділік.

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ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ С КРАЕВЫМИ УСЛОВИЯМИ КАК БИОЛОГИЧЕСКАЯ МОДЕЛЬ

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Аннотация. Рассматривается краевая задача для интегро-дифференциального уравнения. Описывается предложенный Д.С. Джумабаевым метод решения этой задачи и даются комментарии его результатов. Строится новое общее решение интегро-дифференциальных уравнений и устанавливаются его свойства. Строятся алгоритмы для нахождения решений краевых задач для интегро-дифференциальных уравнений и устанавливаются условия их однозначной разрешимости в терминах исходных данных. Результаты иллюстрируются примерами.

Ключевые слова: интегро-дифференциальные уравнения, краевые задачи, новое общее решение, алгоритм, разрешимость.

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