IRSTI 27.01.45

AL-FARABI'S MATHEMATICAL WORLD VIEW AND OVERVIEW OF ROUND BODY RESEARCHES

A.E. ZHUMAGALIEVA[0000-0002-4224-5386], **L.A. KOZHAKHMETOVA**[0000-0002-5002-0785]*
West Kazakstan university after Makhambet Utemisova, Uralsk, Kazakhstan
*e-mail: lyako1990@gmail.com

Abstract. This article examines the mathematical research of Abu Nasir al-Farabi in honor of the 1150th anniversary, with particular emphasis on his research on round bodies. Examples from the research of the great thinker al-Farabi are given and diagrams are given to determine the center of a given circle, how to draw a circle indirectly, how to fill a segment to a full circle. You can see the theorems of the study of modern round bodies, begun in the works of Al-Farabi. Parts of the Center In the work "Mathematical Treatises" describe the center of a circle, perpendicular to the perpendicular radius of the circle, the method of dividing the arc into three equal parts and a diagram of the method of drawing a house or a sphere, obtained in two equal proportions, relative to another house or ball. Examples are given in comparison with Euclid's research on round bodies and Al-Farabi's research. The article proves the similarities and differences between Euclid's drawings and those of Al-Farabi. Al-Farabi's mathematical treatise contains 15 illustrations, each of which has a clear explanation. This explains why drawings can be studied and used in everyday life. These drawings prove once again that Al-Farabi was a very thoughtful scientist.

Key words: Al-Farabi, research, circle, center, segment, point, circle, arc.

Introduction. Abu Nasir al-Farabi, a great thinker of the East, was born in 870 in the city of Farab, now called Otrar, at the confluence of the Arys and Syr rivers (a medieval city near the modern city of Otrar in the South Kazakhstan region). Farabi's full name is Abu Nasir Muhammad Ibi Muhammad ibn Uzlag ibn Tarhan Al-Farabi. We know that Al-Farabi came from a wealthy Turkic tribe, as evidenced by the fact that his full name is "Tarkhan". The Arabs called Otrar, the ancient Kazakh city of birth, Barba-Farab, and from there he came from Abu Nasir al-Farabi, that is, from Faraban.

At the same time, according to the records of the survivors, the city of Otrar in the IX century is one of the most important places in terms of historical relations and trade routes. Problems of geometry and some other branches of mathematics were closely connected with the development of the theory of historical geometric constructions. Euclidean geometry, founded in 300 BC, states that "a straight line can be drawn from any point to any point", "a finite line can be drawn as long as necessary (infinite)", "a circle can be drawn from any center to any size" , etc. The axioms show the importance of the role of constructions in the formation of geometry [2].

Geometric constructions IX - XV centuries. Al-Farabi's work "Spiritual manipulation of the secrets of nature through geometric figures" was devoted to the problems of geometry, and about 150 drawings were published. Fifteen problems are solved with a ruler and a compass with a constant pitch. Al-Farabi's main achievement was the collection and systematization of materials on the problems of geometric construction, scattered everywhere, and the assignment of "principles" that made him a certain branch of geometry [5].

The main part. Al-Farabi's manuscript does not mention the definition of the circle center in his first book. This report is considered in the treatise of Abu al-Wafi.

How to fill the segment to the full circle, then we put the segment ABC and divide it at point B. Draw the lines AB and BC and construct the right angles BCD and BAD on each of the points A and C on the lines AB and BC. Draw a line BD and divide it at point E. Then the point E is the center of the arc ABC.

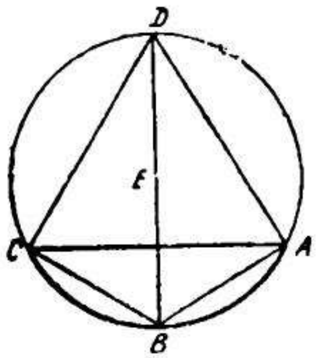

Figure 1 - **Indirect driving**

How can we make a tangent from point A to the circle BC with center D, then we draw the line AD. It intersects the circle BC at point B. Draw a circle AE at a distance DA from the center D. Construct a right angle ABE at point B and draw a line ED that intersects the circle BC at point C. Let's add A and C. In this case, the AC is carried out in the circuit of the indirect BC. Euclid corresponds to the chapter in the 17th appendix of the III book "Nachala". Here is his picture [1, 4].

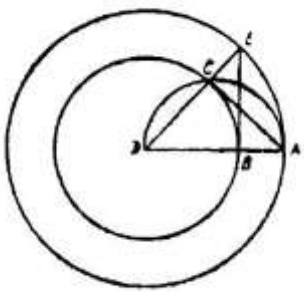

Figure 2 - **Indirect conduction in Euclid's book "Beginning"**

Mathematical substantiation: The tangent to the circle is perpendicular to the radius.

If it is indirect, according to the method of the craftsman, then we place the ruler on the line BC and open the compass to one dimension; If one end of it moves along the ruler, then the other end passes through the point A and gives a line parallel to BC. Al-Farabi demonstrated the method of drawing using a ruler and compass (Figure 3).

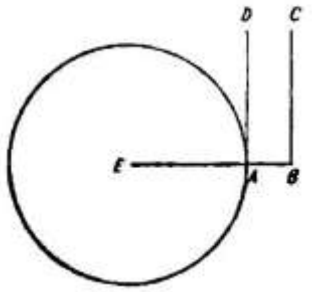

Figure 3 - **The method of drawing with a ruler and compass**

If we draw a tangent from point A to the circle of the wheel ABC, then we connect point A with the center of the circle, point D, so we draw A and D [AD]. Construct a rectangle DAE along the line AD at point A. Then the line AE is adjacent to the wheel ABC. Here is a picture of him. According to Euclid in the 16th appendix of the III book "Nachala" [1,4].

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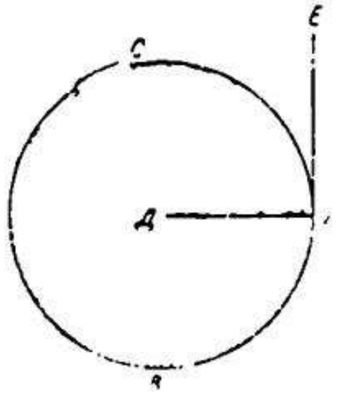

Figure 4 - **Drawing in Appendix 16 to Book III of Euclid "Beginning"**

If the line between the lines AB and AC of the triangle ABC is parallel to BC and draws a line equal to the given D [and if BC is less than the line D], then the line BC in its direction [until point E is equal to [D] if BC is greater than line D, we draw a line BE equal to] D on line BC.

Draw a line parallel to the line AB from the point E. It intersects AC at point G. Draw a line parallel to the line BC from the point G; This is the line GH that intersects AB. Then the line GH is equal to the line D. 34 appendices of the first book of Euclid «Nachala» prove the correctness of the decision [5].

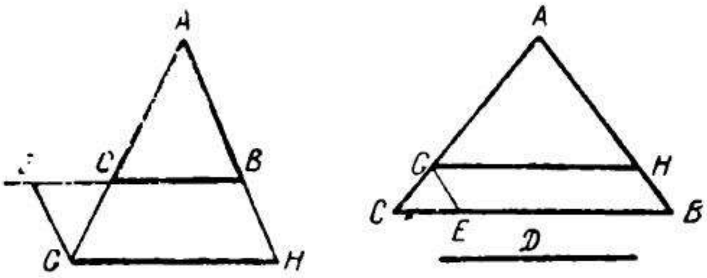

Figure 5 - **Pictures in the works of Al-Farabi and Euclid**

If you want to draw a line between the lines AB and AC of the triangle ABC parallel to the line BC, for example, on the line AB equal to the distance it intersects, ie equal to the line EB, then divide the angle ACB by the line BD and draw the line B from point DC. Then the line DE is equal to the line EB. The solution of the problem is shown in the following figure. The correctness of the construction is proved by the appendices 29, 6 of the first book of Euclid "Nachala" (Figure 6) [5].

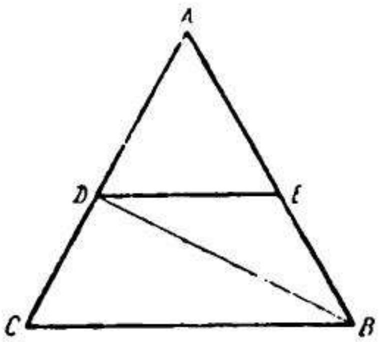

Figure 6 - **Appendix 29, 6 of the first book of Euclid "Beginning**

If, for example, in a triangle ABC it is necessary to draw a line parallel to the line BC and a line DE equal to the lines BE and F. Then draw a line BG equal to the line F, draw a line GH parallel to AB through the point G and divide the angle HGC [GD] and draw a line DE parallel to the line BC from point D. Then the line DE is equal to the lines BE and F (Figure 7) [5].

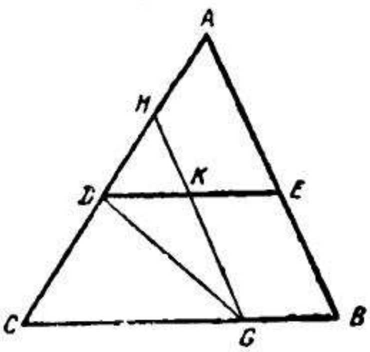

Figure 7 - **The line DE is equal to the lines BE and F.**

About drawing a triangle equal to another triangle. If it is a triangle whose sides are equal to the sides of another triangle [for example, ABC], then we draw a straight line DE and draw a line DG equal to AB, GH equal to BC and HF equal to CA. Let's take the point G as the center and give the part of the circle at a distance GD, so let's take the point H as the center and give the part of the circle at the distance HF. The first part [the second part] intersects at point I. Next, let's draw lines GI and IH. Then the sides of the triangle GIH are equal to the sides of the triangle ABC. The solution is shown in the figure below [5].

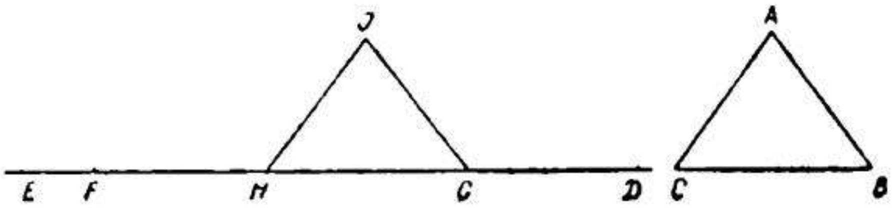

Figure 8 - **The sides of the triangle GIH are equal to the sides of the triangle ABC.**

On the division of an angle into three equal parts. If he divides the angle ABC into three equal parts, then the angle ABC is a rectangle, we draw an equilateral triangle DBC on the line BC. Then the angle ABD is one third of the rectangle. Divide the DBC angle. Each angle of an isosceles triangle is equal to two-thirds of the right angle (Figure 9) [5].

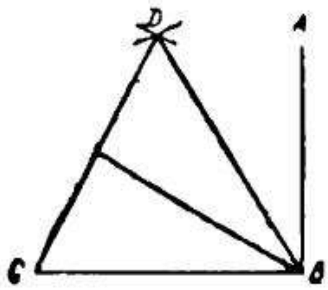

Figure 9 - **Each angle of an equilateral triangle is equal to two-thirds of the rectangle**

If the angle is smaller than the right angle, then we take the point B as the center and draw a circle DAC at a distance BA. Place BD at right angles to BC and draw CB to point E until it intersects the circle. Bring the ruler to point A and move it along the circumference of the wheel CDE until the line HF between the perpendicular DB and the arc DE is equal to the line DB, in which case the ruler does not deviate from point A. Then we draw an arc EK equal to the arc EF and connect KB in the direction to the point L. Then the angle LBC is one third of the angle ABC. Then divide the angle ABL (Figure 10) [1].

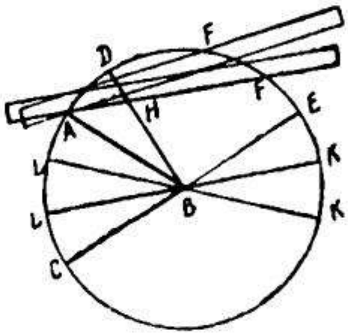

Figure 10 - **The angle LBC is one third of the angle ABC**

Another way to divide an angle into three equal parts. Let's construct an acute angle ABC and if we want to divide it into three equal parts, we will draw a perpendicular from point A [from a point on the line BC]. Bring the ruler to point B and move until the line between the lines AD and AH is doubled AB. For example, the line DEB, and therefore the line DE is a double line AB. Therefore, the angle DBC is one third of the angle ABC. Here is a picture of him [5].

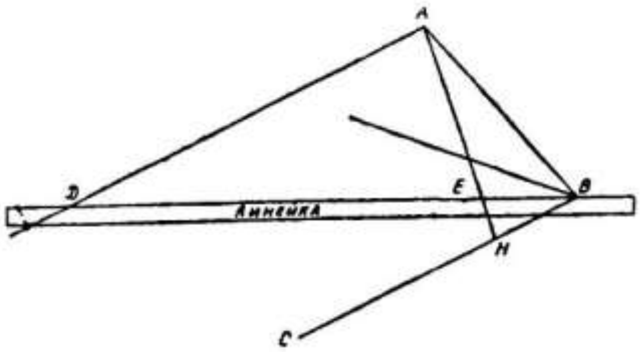

Figure 11 - **Divide the angle into three equal parts**

On the division of the arc into three, equal parts. If he wants to divide the arc ABD into three equal parts, then we find the center of the circle where the arc is located. Let this be the point E. Adding A and E, E and D, we divide the angle AED into three equal parts by the lines EV and EC, which intersect the arc ABCD at points B and C. Then the arc ABCD is divided into three equal parts arcs AB, BC and CD (Figure 12) [5].

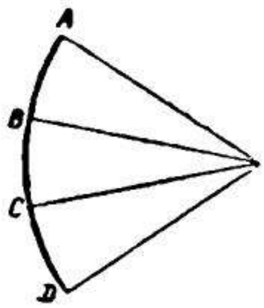

Figure 12 - **Divide the arc ABCD into three equal parts arcs AB, BC and CD**

Consider a report on building a house or balloon that is two times larger than the house or balloon or in relation to the other. If it is necessary to build a square house, which is a double house of equal length, width and height, or to build a double ball, or to divide a ball or a ball of equal diameter, and how to put a ball of the same length and diameter? Construct an equal line AB, draw a line AC on the line AB along the double rectangle and complete the plane figure DABC.

Draw diagonals AD and BC. They are separated at point F. We draw DC and DB lines in their direction. We place the edge of the ruler at point A and move it along the lines GC and EB [until they intersect at points E and G] so that GF and FE are equal. Then the length of the house or the diameter of the sphere is a line BE (Figure 13) [5].

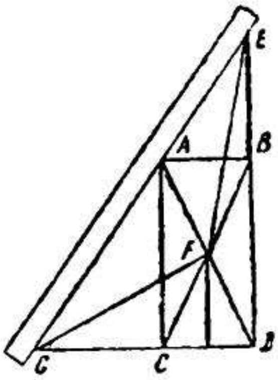

Figure 13 - **The length of the house or the diameter of the ball**

About making a burning mirror. If we want to make a mirror that burns an object from a distance with the help of sunlight, then first we need to make a ready-made model that defines the mirror (pattern). Let's make a circle, half the diameter of which is equal to the distance of the object we want to burn. This is the ABC circle. Let's draw its diameter ADC. Let's draw some equal segments from point C on the line DC. The smaller these sections, the better and more accurate the finished sample. These sections are CF, FH, HG, GE and ED. Let's draw lines E, G, H and F at right angles through the point D [to CD] and extend them on both sides to the points B, I, K, L and M. Let's add points C and B, C and I, C and K, C and L, C and M. Let us draw the line FN equal to the line CM, the line HX equal to CL, the line GO equal to CK, the line EP equal to CI and the line DS equal to CB. Let's add points C, N, X, O, P and S and draw a pattern along the line. Let's make a mirror out of a lot of metal, such as iron, bronze, copper or zinc, and if possible, polish it. If the mirror is crooked, we adjust it according to the finished pattern by placing the finished sample on the mirror so that the point C coincides with the center of the finished sample on the mirror. Then we get a combustible mirror with a high ignition power (Figure 14) [5].

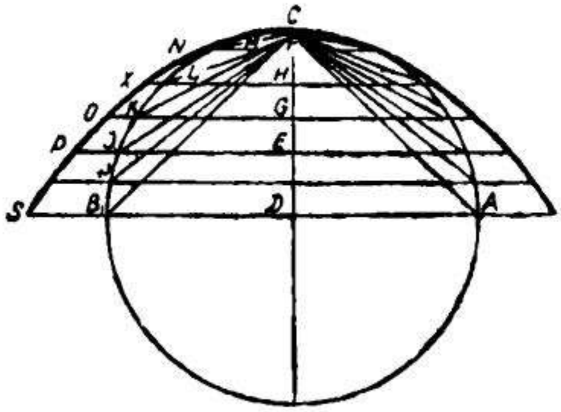

Figure 14 - **Drawing of a combustible mirror with high flammability**

The second way to make a burning mirror. If we want to draw it, we can take any distance, half of which is the line AB, and draw it in its direction to the point C. Let us place the line DB perpendicular to point B on both sides and draw small lines BE, EG, GH and HC equal to each other on the line BC. Divide AE at the point F and draw a circle at a distance FA from the center F. It intersects the line BD at points I. Draw lines IL parallel to the lines AC from points I and a line parallel to the line BD from points E to the points L. Divide the line AG at the point M and draw a circle at a distance MA from the point M. It intersects the line BD at N points. Draw lines NX parallel to the line AC from points N to the point X. Divide the line AN at the point O and draw a circle at a distance OA from the center O. It intersects the line BD at points P. Draw lines from points P to points Z parallel to BC. Connect the points B, L, X and Z with a line and get a sample. If we are going to check the finished sample, we place it at point B in the middle of the mirror. Thus, we can obtain a combustible mirror with a high ignition power (Figure 15) [5].

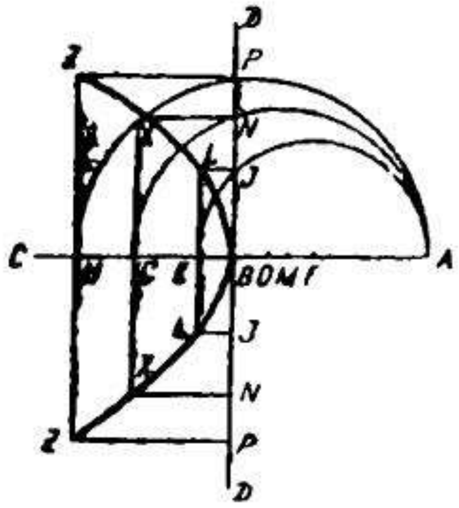

Figure 15 - **Ignition mirror diagram**

Conclusion. Let us now briefly consider the content of Farabi's mathematical treatises contained in the above collection. We have already mentioned that Farabi in his famous "Encyclopedia" defined the subject and content of mathematical sciences, as well as other sciences. Farabi focuses on each of the chapters of mathematics and defines the disciplines, the content, which is not a small glove for the development of mathematics. His views on the seventh chapter of mathematics, that is, on the science of manipulation, are especially important here. The ancient Greeks as mechanics knew the project of this science. "Mechanics" means "cunning" in Greek. However, Farabi expanded the content of this science and made it a special branch of mathematics. In terms of content, this field is close to the mathematics (applied mathematics) used in modern practice. The great scientist has a great place in the history of mathematical sciences and technology. One of his great principles in science is the ability to apply mathematics to the study of natural phenomena, to practice.

The main purpose of the science of trickery is to make the possibility a reality. "The science of manipulation is the study of how to find the methods and mechanisms that are necessary to apply mathematical, principled, natural and perceptible bodies deliberately," he said. According to Farabi, the mathematical basis of architecture, geodesy, metrology, artisanship, various tools and other practical areas lies in science [7].

The great scientist, in other words, says that the application of mathematics is innumerable, it is only necessary to be able to find the source, the problem, that is, the trick. Modern science has fully proved that this principle is correct.

References

1. Аль-Фараби. Математические трактаты / Аль-Фараби.- Алма-Ата: Наука, 1972.- 321 б.
2. Аль-Фараби Ә.Н. Он томдық шығармалар жинағы. 3-ші т. Ғылым (математика және жаратылыстану) / Ә.Н. АльФараби.- Астана: Лотос-Астана, 2007.- 304 б.
3. Жарықбаев Қ.Б. Әбу Насыр әл-Фараби. (870-950):библиографиялық көрсеткіш [Мәтін]. /Қ.Б. Жарықбаев. – Алматы, 2012. - 474 б.
4. Мордухай-Болтовского В.В. Начала Евклида / В.В. Мордухай-Болтовского – Москва : ГИЛТТ Москва – Ленинград, 1948. - 403 б.
5. Әл-Фараби - 1150: Ойшыл-математик, жаратылыстанушы, педагог заманауи білім беруде. 3 маусым 2020 ж. [Электрондық ресурс] – Кіру режимі: https://www.kaznpu.kz/kz/12167/press/.
6. Әл-Фараби және оның рухани мұрасы. [Электрондық ресурс] – Кіру режимі: http://elibrary.kaznu.kz/kk/node/5.
7. Әбу Насыр Әл-Фараби ғалымдардың көзімен. [Электрондық ресурс] – Кіру режимі: https://www.uniface.kz/index.php/questions/index.php?post=article&section=1&id=743

References

1. Farabi A. (1972). Matematicheskie traktaty [Mathematical treatises]. Almaty: Nauka [in Russian].
2. Farabi A. (2007). On tomdyk shigarmalar zhinagy, 3-tom [A ten volume collection of works, 3 rd t]. Astana: Lotos-Astana [in Kazakh].
3. Zharykbayev K.B. Abu Nasyr al-Farabi (870-950) bibliografialyk korsetkish. (2007). [Abu Nasyr al-Farabi (870-950) bibliographic index]. Almaty [in Kazakh].

4. Morduhai-Boltovskogo V.V. (1948). Nachala Evklida [The beginnings of Euclid]. Moskva: GILTT Moskva-Leningrad [in Russian].

5. Al-Farabi – 1150: Oishyl – matematik, zharatylystanushy, pedagog zamanayi bilim berude. 03 mausym 2020 zh. [Thinker-mathematician, naturalist, teacher provides modern education]. www.alfarabi.kaznpu.kz. Retrieved from https://www.kaznpu.kz/kz/12167/press/. [in Kazakh]

6. Al-Farabi zhane onyn ruhani murasy.[Al-Farabi and his spiritual heritage] www.kaznu.kz. Retrieved from http://elibrary.kaznu.kz/kk/node/5. [in Kazakh]

7. Abu Nasir Al-Farabi galymdardin kozimen. [Abu Nasir Al-Farabi through the eyes of scientists.] www.uniface.kz. Retrieved from https://www.uniface.kz/index.php/questions/index.php?post=article&section=1&id=743 [in Kazakh]

ӘЛ-ФАРАБИДІҢ МАТЕМАТИКАЛЫҚ ДҮНИЕТАНЫМЫ ЖӘНЕ ДӨҢГЕЛЕК ДЕНЕЛЕР ТУРАЛЫ ЗЕРТТЕУЛЕРІНЕ ШОЛУ

А. Е. ЖУМАГАЛИЕВА, Л.А.КОЖАХМЕТОВА*

М. Өтемісов атындағы Батыс Қазақстан университеті, Орал, Қазақстан

*e-mail: lyako1990@gmail.com

Аңдатпа. Ұсынылып отырған мақалада Әбу Насыр Әл-Фарабидің 1150 жылдығына орай математикалық зерттеулеріне шолу жасалып, соның ішінде дөңгелек денелерге қатысты зерттеулеріне аса көңіл бөлінген. Берілген шеңбердің центрін анықтау, шеңберге қалай жанама жүргізуге болады, сегментті толық шеңберге дейін қалай толықтыру керек деген сұрақтарға ұлы ойшыл әл-Фарабидің зерттеулерінен мысалдар келтіріліп, сызбалары ұсынылған. Әл-Фарабидің еңбектерінен бастау алған қазіргі дөңгелек денелер туралы зерттеулер теоремаларды көруге болады. Математикалық трактат еңбегіндегі Центрдің бөліктері, шеңбер центрін анықтау туралы, шеңберге жүргізілген жанама радиусқа перпендикуляр жайында, доғаны тең үш бөлікке бөлу әдісі және екі еселенген басқа үйге немесе шарға тең қатынаста алынған үй немесе шар салу туралы әдістерінің сызбасы көрсетіліп, түсіндірілген. Евклидтен бастау алған дөңгелек денелер туралы зерттеулері мен Әл-Фарабидің зерттеулерімен салыстырылып, мысалдар келтірілген. Мақалада Евклид сызбалары мен Аль-Фараби сызбаларының ұқсастықтары дәлелденіп, ерекшеліктері анықты көрсетілген. Әл-Фарабидің математикалық трактатынан 15 сурет алынып, әр сызығын анықты етіп түсіндірілген. Ол сызбалардың не үшін зерттеп, неге пайдалануға болатыныда, тұрмыстық өмірде пайдасы да айтылған. Бұл сызбалардан Әл-Фарабидің аса ойлы ғұлама болғаны тағы бір дәлелденді.

Түйін сөздер: Әл-Фараби, зерттеу, шеңбер, центр, сигмент, нүкте, дөңгелек, доға.

МАТЕМАТИЧЕСКОЕ МИРОВОЗРЕНИЕ АЛЬ-ФАРАБИ И ОБЗОР ИСЛЕДОВАНИЙ КРУГЛЫХ ТЕЛ

А. Е. ЖУМАГАЛИЕВА, Л.А.КОЖАХМЕТОВА*

Западно- Казахстанский университет имени Махамбета Утемисова, Уральск, Казахстан

*e-mail: lyako1990@gmail.com

Аннотация. В данной статье рассматривается математическое исследование Абу Насира аль-Фараби в честь 1150-летия, с особым акцентом на его исследования круглых тел. Приведены примеры из исследований великого мыслителя аль-Фараби и даны диаграммы, чтобы определить центр данного круга, как нарисовать круг косвенно, как заполнить отрезок до полного круга. Можно увидеть теоремы исследования современных круглых тел, начатые еще в трудах Аль-Фараби. Части Центра В труде «Математические трактаты» описывают центр круга, перпендикуляр к радиусу перпендикуляра круга, метод деления дуги на три равные части и схему метода рисования дома или сферы, полученной в двух равных пропорциях, относительно другого дома или шара. Приведены примеры в сравнении с исследованиями Евклида о круглых телах и исследованиями Аль-Фараби. В статье доказываются сходства и различия рисунков Евклида и рисунков Аль-Фараби. Математический трактат Аль-Фараби содержит 15 иллюстраций, каждая из которых имеет четкое объяснение. Это объясняет, почему рисунки можно изучать и использовать в повседневной жизни. Эти рисунки еще раз доказывают, что Аль-Фараби был очень вдумчивым ученым.

Ключевые слова: Аль-Фараби, иследование, круг, центр, отрезок, точка, круг, дуга.